

# IMPROVEMENT OF MICRONETWORK ACCURACY BY INVOLVEMENT OF ADJUSTMENT WITH WEIGHT

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## الخلاصة:

لقد ساعد التطور التكنولوجي السريع في اجهزة الحاسوب وفي اجهزة المسح في التغلب على مشكلة تحسين وتقييم دقة الشبكات المصغرة للاغراض الهندسية الدقيقة. وهذا البحث تعامل في حل مشكلة اسلوب القياس المختلط من اجل تحسين دقة هذه الشبكات. ان دراسة مقادير الاخطاء والعلاقة بينها من الامور المهمة في مراحل التصميم والقياس والتصحيح. وقد توصل البحث الى ان استخدام الاوزان في القياس المختلط من الامور المهمة التي تساعد في تحسين دقة الشبكات المصغرة وهذه الاوزان تم تحديدها عن طريق تقييم وايجاد الخطا المعياري لكل نوع من انواع القياس.

## ABSTRACT

The rapid development in computer Technology and in the instruments used in field measurements helped to overcome the problem of accuracy improvement and accuracy assessment of high precision networks. This paper deals with solution of the use of mixed observation systems, it is an essential requirement to improve the accuracy. The study of the magnitude of the correlation of error is of great importance in the efficient performance of planning, measuring and adjusting operation of survey. This paper concludes that the use of weights for mixed observation constitutes an important part in the improvement of the accuracy of a micronetwork. Weights which are determined for mixed observation after the assessment of the standard error of unit weight for each kind of observation .

## **INTRODUCTION**

Priori to any surveying project, engineers have to estimate and determine the accuracy of surveying. Estimation of accuracy should be continuous with the field measurements and adjustment. Because of economical reason, field measurements have to be as few as possible. Hence an estimate of the accuracy of the surveying is required. For this reasons, engineers cannot say that they perfectly know values or accuracy in advance.

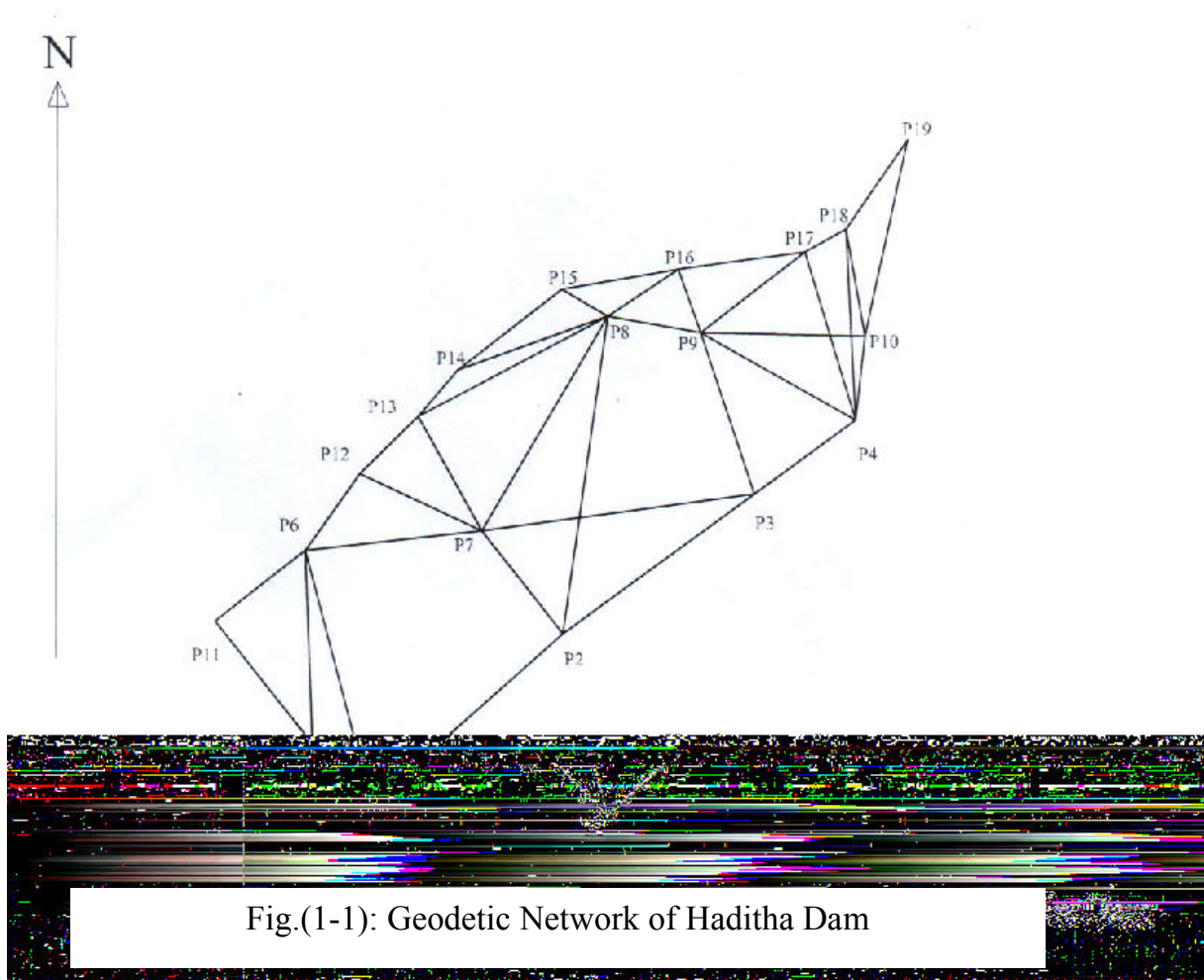
The problem is to assign weights to separate observations and to combine them, because assigning weights can not be done according to a fixed rule but is related to many variables such as the accuracy of instruments used, strength of figures, atmospheric conditions, and ..... etc.

In this paper the above mentioned problems are treated by means of the variance covariance matrix. This matrix can be computed for any network irrespective of whether or not the actual observations have been carried out. The variance covariance matrix is used to determine the standard errors of the position of the stations and the accuracy of the observed quantities. After adjustment, the mean square errors of unit weight of the trilateration and triangulation methods have been determined. Then these values are used to determine weights for mixed observations. The network of Haditha Dam shown in Fig. ( 1-1 ) is used for this purpose .

## **1- Basic Principles in Surveying**

### **1-1 Horizontal Control**

The Horizontal positions of points in a network are developed to provide accurate control for subsidiary surveys. These positions can be obtained in different ways or methods in addition to traversing. Such as triangulation, intersection and resection. Since the advent of long range EDM's, a method of surveying called trilateration has been adopted and combined with triangulation in order to increase the strength of the network.



## 1- 2 Precautionary Suggestions

There are many precautions that have to be fulfilled , because lack of action may prevent the attainment of the desired accuracy for project , even if all major requirements have been met .

The requirements specify that the micrometer must be brought into coincidence twice , and both reading are recorded for all theodolites , then the mean of measurements is used to determine the direction . The measurements of each set should be made as quickly as possible . Sequence of pointing at two targets should follow in each set and observing with the telescope in both the F.L and F.R positions . Frequent checking of leveling and centering and changing the circle setting for each set is a must . A final setting of the pointing using the target screw should be

terminated against the pressure of the spring . The following procedures are precautionary suggestions rather than part of the formed specifications.

### **1-3 Collimation and Eccentricity**

These are several problems that affect the accuracy of both angle, distance measurements and targets . Instruments and targets can be positioned within about 1 mm of the true center. An instrument or target may be deliberately or accidentally operated or observed in an eccentric position , which is usually a short distance from the true station. Accurate measurements of distance involving the eccentric and true points must be obtained in order to reduce the observations to the true point or to maintain the relationship.

### **1-4 Criteria of Adjustment**

The most generalized criterion used in adjustment are :

- 1- The magnitude of the adjustment “ improperly called correction “ of the observed variable is a minimum .
- 2- The magnitude of these adjustment are commensurate the respective accuracies or with respective relation weights of the observed variables .
- 3- The adjusted variables (observed and unknowns) fulfil the constraints imposed by the survey of the geometrical configuration .

The statement 1 and 2 are mathematically condensed in : the sum of the observed variable adjustment squared times their observed variable respective weight are a minimum. That is why this adjustment criterion is known as the “Least Squares principles“. The variation of coordinates method may be used to adjust any combination of direction, angle and distance . The method consists of computing provisional ( approximate ) coordinates for each station and then setting up equations one for each observation, which relate the unknown errors in these coordinates to the difference between observed quantities and quantities computed from the coordinates . The repetitive process involved in setting up these equations is extremely easy to handle automatically, so the method is practically for use with a digital technique.

### ***2: Observation Equations for Horizontal Angles and Distance for Hiditha Dam***

The application of least squares adjustment in plane coordinate surveys includes formulation and linearization of the equation (distance, angles ) encountered in the adjustment of plane coordinates by the method of indirect observations ( variation of coordinates ), least squares position adjustment for a typical procedure employed in plane coordinate surveying as discussed below.

The observation of the measured angle as shown in Fig. (1-2) can be expressed as :

$$\alpha + V\alpha = aPB - aPA = \tan^{-1}\left(\frac{X_B - X_P}{Y_B - Y_P}\right) - \tan^{-1}\left(\frac{X_A - X_P}{Y_A - Y_P}\right) \dots\dots\dots (1-1)$$

where:

aPB: the azimuth of line PB.

aPA: the azimuth of line PA.

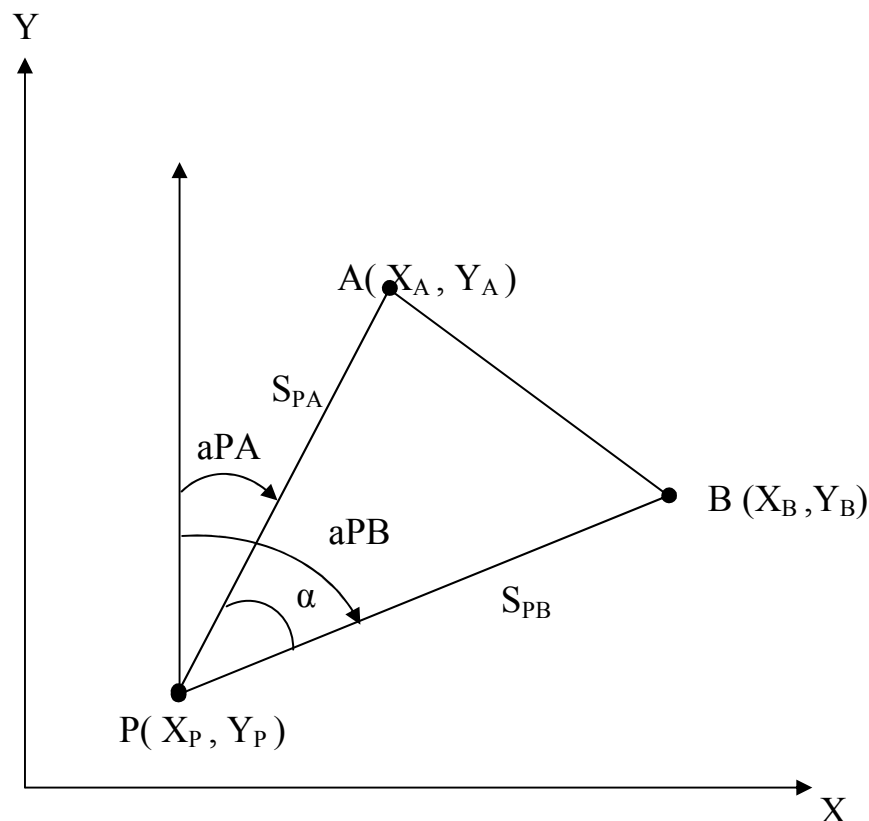


Fig.(1-2) Interrelation of horizontal angles, distances, azimuths and (x,y) coordinates in plane surveying.

The above equation is non linear it can be made linear by Taylors method and the final form of equation (1-1) is given as follows [12] and [5].

$$\begin{aligned}
 V\alpha + \alpha &= \tan^{-1} \left( \frac{XB_0 - XP_0}{YB_0 - YP_0} \right) - \tan^{-1} \left( \frac{XA_0 - XP_0}{YA_0 - YP_0} \right) \\
 &= \int^g \left[ \frac{\cos(aPB_0)}{SPB_0} \Delta XB - \frac{\sin(aPB_0)}{SPB_0} \Delta YB + \frac{\cos(aPA_0)}{SPA_0} \Delta XA \right. \\
 &\quad \left. - \frac{\sin(aPA_0)}{SPA_0} \Delta YA + \left[ \frac{\cos(aPA_0)}{SPA_0} - \frac{\cos(aPB_0)}{SPB_0} \right] \Delta XP \right. \\
 &\quad \left. + \left[ \frac{\sin(aPB_0)}{SPB_0} - \frac{\sin(aPA_0)}{SPA_0} \right] \Delta YP \right] \dots\dots\dots (1-2)
 \end{aligned}$$

where:

$\int^g$  : a coefficient to convert radians to grad 1= 63.6620.

index 0 : indicates an approximate value of the corresponding quantity .

Substituting  $b_a, a_1, a_2, \dots, \ell$  the corresponding constants , the Eq. ( 1 - 2 )

$$a_1 \Delta X_B + a_2 \Delta Y_B + a_3 \Delta X_A + a_4 \Delta Y_A + a_5 \Delta X_P + a_6 \Delta Y = \ell + V_\alpha \dots (1-3)$$

where  $\ell$  column matrix.

the constants  $a_1, a_2, \dots, a_6$  are computed using approximate values Eq. (1 - 2)

The observation equation of a measured distance  $S_{PA}$  as shown in Fig. (1 - 2)

$$SPA + VPA = \sqrt{(X_A - X_P)^2 + (Y_A - Y_P)^2} \dots\dots\dots (1-4)$$

The linear form of Eq. ( 1 - 4 ) is given as follows [ 5 ] and [ 12 ]

$$\begin{aligned}
 S_P = S_{PA_0} + v_{SPA} &= \sin(aPA_0) \Delta X_A + \cos(aPA_0) \Delta Y_A \\
 &\quad - \sin(aPA_0) \Delta X_P - \cos(aPA_0) \Delta Y_P \dots\dots\dots(1-5)
 \end{aligned}$$

Substituting  $b_1, b_2, \dots, \ell$  the corresponding constants , then Eq. ( 1 - 5 ) might be written as follows :

$$b_1 \Delta X_A + b_2 \Delta Y_A + b_3 \Delta X_P + b_4 \Delta Y_P = \ell + V_{SPA} \dots\dots (1-6)$$

the constants  $b_1, b_2, \dots, b_4$  are computed using approximate values Eq. ( 1 - 5 )

For geodetic monitoring of Haditha Dam the side P1 P5 is a base line and an approximated values was computed by forward computation from point P by using the following formula:

$$X_{pi} = X_{pi-1} + S \cos \alpha \dots\dots\dots (1-7)$$

$$Y_{pi} = Y_{pi-1} + S \sin \alpha \dots\dots\dots (1-8)$$

Table (1-1) contains an approximate coordinates.

By using the an approximate coordinate , (63) observation equations can be formed for angles and (39) observation equations can be formed for distances after substituting in equations (1-2) and (1-5) consequently .

By using matrix algebra , a set of observation equations may be written in the short formula as [12].

$$A X = L + V \dots\dots\dots (1-9)$$

Where :

X = the column vector of parameters .

A = the matrix of coefficient .

L = the column correction ( residual )

The least squares solution for finding the vector matrix X is obtained by applying the condition  $v^T v = \text{minimum}$  . If the observation equations are not of the same accuracy , it should be introduce the weight matrix P and the condition is ( in the matrix form ):

$$V^T P V = \text{min} \dots\dots\dots (1-10)$$

The above condition , if applied in equation (1-9) gives the so-called normal equations [4].

$$(A^T P A) X + A^T P L = 0 \dots\dots\dots (1-11)$$

where :

$A^T P A$  : N is a square and symmetrical matrix .

The number of normal equation is equal to the number of unknowns , therefor , a unique solution for the unknown is now possible in the form [4].

$$X = - (A^T P A)^{-1} A^T P L = - N^{-1} A^T P L \dots\dots\dots (1-12)$$

The above procedure of the adjustment is based on the assumption that the correction.  $\Delta X$  and  $\Delta Y$  are differ in small amount.

Table (1 –1): Approximate coordinates of Geodetic Network of Haditha Dam

Position	N( m )	E ( m )
P1	3794018.448	256878.987
P2	3795415.511	257524.028
P3	3796543.033	259116.429
P4	3797149.447	260006.365
P5	3794493.946	255373.016
P6	3796100.558	255305.343
P7	3796250.899	256823.498
P8	3798003.119	257877.435
P9	3797860.448	258653.036
P10	3797827.536	260066.043
P11	3795533.400	254544.569
P12	3796717.008	255754.756
P13	3797196.653	256239.274
P14	3797541.144	256581.746
P15	3798201.440	257469.563
P16	3798339.122	258466.789
P17	3798496.923	259551.330
P18	3798681.707	559906.250
P19	3799327.269	266550.487

If the approximate coordinates , obtained from preliminary calculations , differ considerably from the adjusted values , the result of the adjustment will not be corrected. If there is any doubt about adjustment should be repeated , with the



approximate ones . If the first adjustment is corrected , then values  $\Delta X$  and  $\Delta Y$  from the second adjustment should be negligibly small. Sometimes two or more adjustment ( iteration ) may be needed.

The above procedure of adjustment is used for monitoring the network of Haditha dam. Different methods and solutions were performed on a microcomputer using the soft ware LOTUS .

### ***3- Error Analysis of the Adjusted Network by the Variance-Covariance Matrix***

The solution of the least-squares adjustment , as given by Eq. (1-12) , includes the inverse matrix  $(A^T P A)^{-1}$  , which should be identical with the inverse matrix Q obtained from the equation :

$$\text{Cov} ( x ) = \sigma_0^2 ( A^T P A )^{-1} = \sigma_0^2 Q \quad \dots\dots\dots ( 1 - 13 )$$

If the shape and the type of observations in the adjusted network are the same as in the design .

Only question in calculating the Variance-Covariance Matrix after adjustment is whether the matrix Q should be multiplied by the a priori  $\sigma_0^2$  or by a posterior :  $\hat{\sigma}_0^2$  that can be calculated by the equation :

$$\hat{\sigma}_0^2 = \frac{V^T P V}{n - u} \quad \dots\dots\dots ( 1 - 14 )$$

where :

n : number of observation.

u : number of unknowns.

Different authors have a given different answers but it seem clear that if the comparison of  $\hat{\sigma}_0$  and  $\hat{\sigma}_0^2$  passes the Chi- squares test any value of  $\sigma_0^2$  that is within the obtained confidence is equally good statistically. But the correct use of  $\sigma_0^2$  in the design stage ( preanalysis network ) therefor, the value of  $\hat{\sigma}_0^2$  may be used.

The calculation of standard deviations of the functions of adjusted coordinates is extremely important to the users of the control network in legal and engineering surveys .

Since the practicing surveyors should be able to estimate the accuracy of the observations which calculated from the given coordinates .

The calculation of the errors of the functions follow the general rule of error propagation [4].

$$\text{Cov} ( L ) = A \text{Cov} ( X ) A^T \dots\dots\dots ( 1 - 15 )$$

The use of the mean square error of unit weight . As a basic procedure in error analysis for different methods :

1. For trilateration  $\sigma_d^{\wedge} = \pm 0.005 \text{ m}$
2. For triangulation  $\sigma_a^{\wedge} = g^{cc} \cong \pm 0.018 \text{ m}$
3. For mixed  $\sigma_m^{\wedge} = 0.007 \text{ m}$

For error analysis of the adjusted network by the use of variance covariance matrix , see the table ( 1 - 2 ) . Obviously the mixed method gives a good result . All the number of redundant measurements in triangulation is larger than that of trilateration , the accuracy of the unknowns in trilateration is found to be superior . See Fig. ( 1 - 3 ) . It is believed that errors in measured angles are due to negligence in the precaution suggested in a section ( 1 - 2 ) and due to poor field procedures .

Table(1-2)the accuracy of the unknowns (resultant error) adjusted by trilateration , triangulation and mixed

Position	trilateration	triangulation	mixed
P2	$\pm 0.032$	$\pm 0.050$	$\pm 0.011$
P3	0.057	0.091	0.020
P4	0.072	0.117	0.025
P6	0.029	0.038	0.010
P7	0.036	0.047	0.012
P8	0.064	0.088	0.021
P9	0.066	0.095	0.022
P10	0.057	0.125	0.027
P11	0.027	0.036	0.009
P12	0.038	0.054	0.013

P13	0.046	0.065	0.015
P14	0.053	0.137	0.018
P15	0.064	0.093	0.021
P16	0.069	0.102	0.024
P17	0.079	0.126	0.028
P18	0.089	0.137	0.029
P19	0.097	0.179	0.034

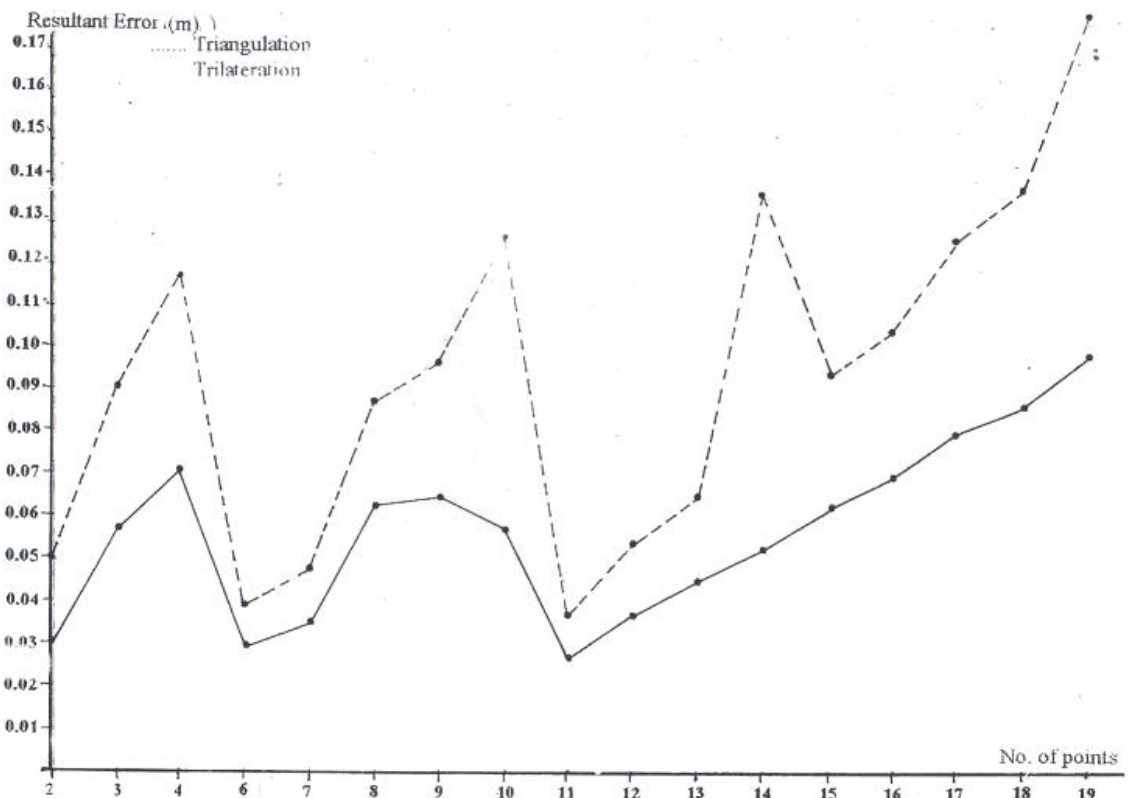


Fig. ( 1-3 ) The position accuracy of the different kinds of adjustment. V - Scale 1:1 V- Scale 1:1

#### 4- Internal Weighting System in the adjustment

The precision of the adjusted observation depends on instruments , skill and the method of adjustment , but the actual values of these distances and angles must be considered too . Therefor , internal weight of the observation in the adjustment by different methods must be considered . [ 4 ] illustrated that the weight of distances equal inverse of square distance (  $\frac{1}{s^2}$  ) but , when used this method does not improve the accuracy of unknowns , and when used a priori weight , the accuracy of the unknowns approximately similar to accuracy of unknowns when  $p = 1$  see table ( 1 – 3 ) . Therefor , in the adjustment of trilateration , there is not need to introduce weight

of all sides are measured with the same instrument.[ 6 ] illustrated that weight of direction relating to one leg of a certain angle is  $S_1$  and that the other  $S_r$  (where  $S$  indicates the length of direction ) then according to the law of the propagation of errors using the formula .

$$\frac{1}{PA} = \frac{1}{S_1} + \frac{1}{S_r} \dots\dots\dots ( 1 - 16 )$$

the weight of the angle is :

$$PA = \frac{S_1 S_r}{S_1 + S_r} \dots\dots\dots ( 1 - 17 )$$

But when the using this method does not improve the accuracy of unknowns [table ( 1- 4)] . Therefor in the adjustment of triangulation , there is no need to introduce weights if all angles are measured with the same instrument and with same precision.

For mixed observation there are two methods of assigning weights the first method used a posteriori weights is (  $P = \frac{1}{\sigma^2}$  ) .By applying Eq. (1- 13 ) . But does not improve the accuracy of the unknowns and give the same resultant when  $p = 1$  as shown in table ( 1 – 5 ) .

The second method used the ratio (  $\sigma_a^2 / \sigma_d^2$  ) ( when the  $\sigma_a$  is the mean square error of the unit weight for triangulation method and the  $\sigma_d^2$  is the mean square error of the unit weight for trilateration method ) was used this ratio was found to be equal to ( 13 / 1 ) . Therefor , this quantity ( 13 / 1 ) was used in the diagonal of P matrix as ( 13 ) for the distances observations and ( 1 ) for the angles observations . By using this method it was found that the accuracy of position are improved when used  $p = 1$  as shown in table ( 1 – 5 ) .

As conclusion to these methods it was found that using weights is more suitable for micro network used in precise engineering projects as illustrated in Fig.( 1-4).

Table ( 1 – 3 ) the position accuracy adjusted by trilateration

$$\text{with } p = \frac{1}{s^2}, P = \frac{1}{\sigma^2}, p = 1$$

Position	Resultant error $p = \frac{1}{s^2}$	Resultant error $P = \frac{1}{\sigma^2}$	Resultant error $P = 1$
2	± 0.068	± 0.035	± 0.031
3	0.079	0.068	0.065
4	0.121	± 0.080	0.070
6	0.051	0.033	0.030
7	0.060	0.038	0.034
8	0.059	0.036	0.049
9	0.111	0.052	0.064
10	0.132	0.074	0.077
11	0.045	0.029	0.027
12	0.108	0.042	0.043
13	0.059	0.049	0.043
14	0.087	0.059	0.053
15	0.104	0.068	0.062
16	0.117	0.076	0.069
17	0.134	0.088	0.080
18	0.143	0.094	0.084
19	0.164	0.110	0.094

Table (1-4) the position accuracy adjusted by trilateration With  $P = 1$  and  $PA = \frac{S_1 S_r}{S_1 + s_r}$

Position	$P = 1$	$PA = \frac{S_1 S_r}{S_1 + s_r}$
2	± 0.049	± 0.053
3	0.087	0.018
4	0.166	0.130
6	0.039	0.046
7	0.048	0.054
8	0.088	0.097
9	0.096	0.105
10	0.126	0.124
11	0.038	0.042
12	0.052	0.059
13	0.064	0.066
14	0.0136	0.146
15	0.094	0.094
16	0.103	0.107
17	0.065	0.131
18	0.136	0.141
19	0.177	0.181

Table ( 1 – 5 ) the position accuracy adjusted by mixed methods with P = 1 and

$$P = \frac{1}{\sigma^2} \text{ , and } p = \sigma_a^2 / \sigma_d^2$$

Position	Resultant error P = 1	Resultant error $P = \frac{1}{\sigma^2}$	Resultant error $p = \sigma_a^2 / \sigma_d^2$
2	0.011	0.011	0.007
3	0.029	0.021	0.015
4	0.025	0.026	0.019
6	0.011	0.016	0.006
7	0.012	0.013	0.007
8	0.021	0.023	0.017
9	0.025	0.027	0.018
10	0.025	0.029	0.022
11	0.009	0.010	0.006
12	0.013	0.013	0.010
13	0.015	0.016	0.012
14	0.015	0.016	0.014
15	0.021	0.023	0.017
16	0.023	0.027	0.019
17	0.028	0.029	0.021
18	0.029	0.030	0.024
19	0.035	0.0361	0.027

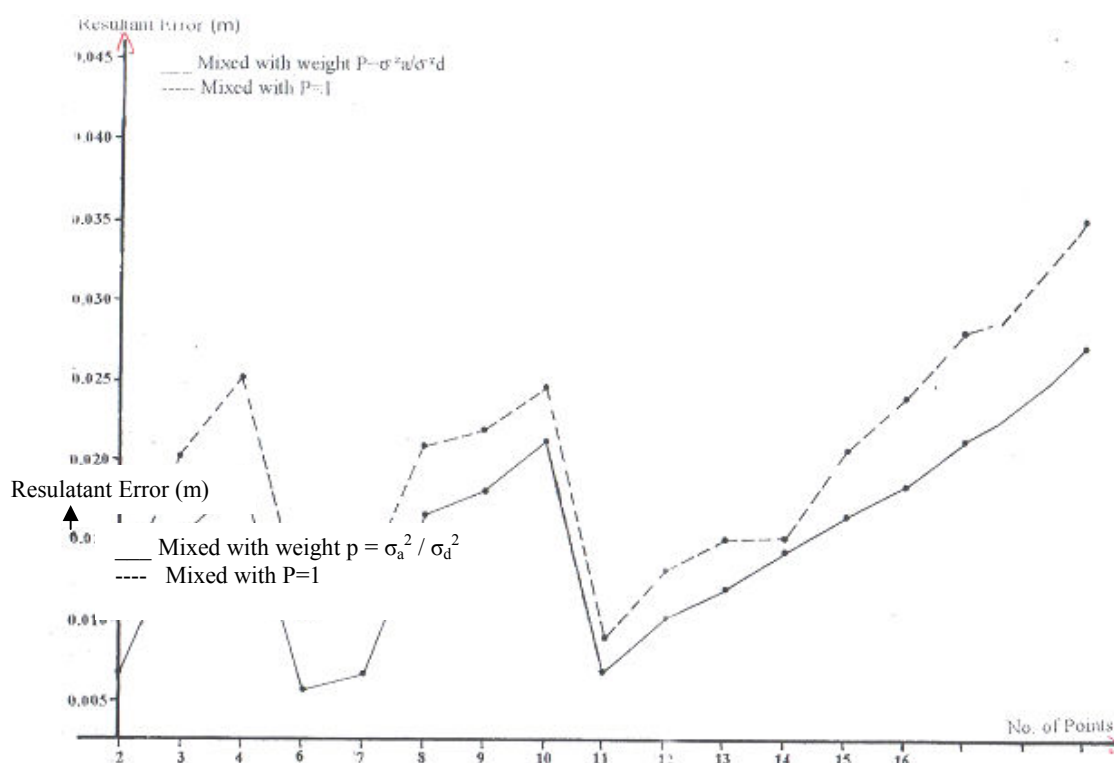


Fig. ( 1-4 ) The position accuracy of the different kinds of adjustment. |V-Scale 4/1

## 5-CONCLUSIONS AND RECOMMENDATIONS

The study and use of accuracy have been obtained through the variance – covariance method by analysing the accuracy of the micronetwork which have been successfully applied to Haditha dam .the following conclusions can be done drawn:

- 1- The use of the mixed observation system is an essential requirement to improve the accuracy of a micro network.
- 2- To improve the accuracy of the unknown positions by giving weights to mixed observation ( $\sigma_a^2 / \sigma_d^2$ ) is found to be effective .
- 3- There are no involved weights for adjustment of triangulation trilateration if all angles or all sides measured with the same instrument and the same precision .
- 4- Safety controls should be distributed outside the structure of the dam at least ( 500 m ) away from the center so that they will not be affected by pressure and tension along the dam.
- 5- The distribution of check points on the dam should regularly distributed .
- 6- The accuracy instruments must be appropriate with the specified accuracy which are needed and the precaution suggested in sec.(1-2) must be considered in the design stage and in the field procedures .
- 7- The actual values of distances and angles must be considered to be very important in order to obtain an optimum value for the weight matrix and to improve the accuracy .

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