Slope Stability Analysis for Sections in Al-Furat River by Finite Element Method

Ahmed H. Abdul Kareem
Lecturer at the College of Engineering at
Anbar university

Abstract
The aim of this study is to analyze the slope stability for sections in Al-Furat River where engineering construction build on it, when a sudden decrease in the river water level happens. Two sections were chosen from the river in the area located about 35 km away from Ramadi city called Tel Aswad where undisturbed samples are taken and laboratory tests are done to obtain the soil parameters which are used in Geo-Slope program. The finite element method was applied in this study with elastic-plastic soil model. The analysis results show that the sections slope chosen from the river are stable. The second purpose of this analysis to reduce the risk of using earth structures when engineering construction build on it. Also, it is clear that the values of factor of safety calculated by the FEM are low compared with limit equilibrium methods.

تحليل استقرارية المنحدرات لمقاطع في نهر الفرات باستخدام طريقة العناصر المحددة

أحمد حازم عبد الكريم
محاضر في قسم الهندسة المدنية / جامعة الانبار

الخلاصة

إن الهدف من هذا البحث هو تحليل مدى استقرارية المنحدرات لمقاطع محددة من نهر الفرات مقامة عليها منشأت هندسية، وذلك عند حصول انخفاض مفاجىء في منسوب النهر. تم اختيار مقطعين من النهر في المنطقة الواقعة على بعد حوالي 35 كيلومتر عن مدينة الرمادي والمعروفة بمنطقة تل اسود. تم في هذا البحث الحصول على عينات غير مخلخلة واجراء الفحوصات المختبرية عليها للحصول على معاملات التربة التي استخدمت في برنامج حاسوب هندسي (Geo-Slope Program). طريقة العناصر المحددة استخدمت في هذا البحث من خلال استخدام الموديل المرن – اللدن للتربة. نتائج التحليل تبين بان منحدرات المقاطع المختارة من النهر مستقرة ، وان اجراء هذا التحليل هو لتقليل مخاطر استعمال هذه المنحدرات عند اقامة المنشأت الهندسية عليها. كذلك تبين بأن قيم معامل الامان المحسوب باستخدام طريقة العناصر المحددة هو اقل مقارنة مع طرق التوازن اللدن

1. Introduction
Stability analysis is an important part of the design of embankments, cut slopes, excavations and dams. In practice, limit equilibrium methods are used in the analysis of slope stability. Failure is considered to occur along an assumed or a known failure surface, and the shear strength required to maintain equilibrium is compared with the available shear strength of the soil.

Most of the limit equilibrium stability methods are two dimensional and assume plane strain conditions. Among these, the methods of slices are the most commonly used, because they can handle complex geometries and variable soil and water conditions.

The finite element method was applied by some authors as Clough and Woodward (1967), Kulhawy and Duncan (1972), Adikari *et al.* (1982) and Veiga Pinto and Neves (1985). [3, 9, 1, 12]

The work presented in this paper, is directed at providing the engineers a general methodology for two dimensional finite element computer programmes to analyze the slope stability for the sections in Al-Furat River.

2. Finite Element Equations

The finite element equation used in the program SIGMA/W which is one of the programmes in the package Geo-Slope formulation for a given time increment is, [7, 8, 13]

$$\int_V [B]^T [C][B] dv \{a\} = b \int_V \langle N \rangle^T dv + p \int_A \langle N \rangle^T dA + \{Fn\} \quad (1)$$

where:
$[B]$ = strain-displacement matrix
$[C]$ = constitutive matrix
$\{a\}$ = column vector of nodal incremental *x*- and *y*-displacements
A = area along the boundary of an element
V = volume of an element
b = unit body force intensity
$\langle N \rangle$ = row vector of interpolating functions
p = incremental surface pressure
$\{Fn\}$ = concentrated nodal incremental loads

Summation of this equation over all elements is implied.

For a two-dimensional plane strain analysis, SIGMA/W considers all elements to be of unit thickness. For constant element thickness, t, Equation 1 can be written as:

$$t \int_A [B]^T [C][B] dA \{a\} = bt \int_A \langle N \rangle^T dA + pt \int_L \langle N \rangle^T dL \quad (2)$$

In an abbreviated form, the finite element equation is

$$[K]\{a\} = \{F\} = \{Fb\} + \{Fs\} + \{Fn\} \quad (3)$$

where:
$[K]$ = element characteristic (or stiffness) matrix
$= t \int_A \left([B]^T [C][B]\right) dA$
$\{a\}$ = nodal incremental displacements
$\{F\}$ = applied nodal incremental force which is made up of the following:
$\{Fb\}$ = incremental body forces
$\{Fs\}$ = force due to surface boundary incremental pressures
$= pt \int_L \left(\langle N \rangle^T\right) dL$, for two-dimensional analysis
$\{Fn\}$ = concentrated nodal incremental forces

SIGMA/W solves this finite element equation for each time step to obtain incremental displacements and calculates the resultant incremental stresses and strains.

3. Strain-Displacement Matrix

SIGMA/W uses engineering shear strain in defining the strain vector [8, 11]

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \end{Bmatrix} \quad (4)$$

The field variable of a stress/deformation problem is displacement which is related to the strain vector through:

$$\{\varepsilon\} = [B]\begin{Bmatrix} \mu \\ \nu \end{Bmatrix} \tag{5}$$

where:
$[B]$ = strain matrix,
u, v = nodal displacement in x- and y-directions, respectively.

SIGMA/W is restricted to performing infinitesimal strain analyses. For a two-dimensional plane strain problem, ε_z is zero and the strain matrix is defined as:

$$[B] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \cdots & \frac{\partial N_8}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & \cdots & 0 & \frac{\partial N_8}{\partial y} \\ 0 & 0 & \cdots & 0 & 0 \\ \frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & \cdots & \frac{\partial N_8}{\partial x} & \frac{\partial N_8}{\partial y} \end{bmatrix} \tag{6}$$

4. Elastic Constitutive Relationship

Stresses are related to strains as follows, within the theory of elasticity, [8,13]

$$\{\sigma\} = [C]\{\varepsilon\} \tag{7}$$

where $[C]$ is the constitutive (element property) matrix and is given by:

$$[C] = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & \nu & 0 \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \tag{8}$$

where:
E = Young's modulus
v = Poisson's ratio

5. The Elastic-Plastic Soil Model

The elastic-plastic constitutive soil model is a very attractive model because it is so simple and easy to understand. The soil is deemed to behave in a linear-elastic fashion up to the point where it reaches its strength and after that the soil is deemed to be perfectly plastic as shown in Fig. 1 [11]. Describing this soil model requires only four common parameters. These are the elastic modulus E, the Poisson's ratio, v, the friction angle (ϕ) and the cohesion (c). The parameters that are used in the study are shown in Table 1. Values of c and ϕ were obtained from laboratory testing while E and v were assumed depending on

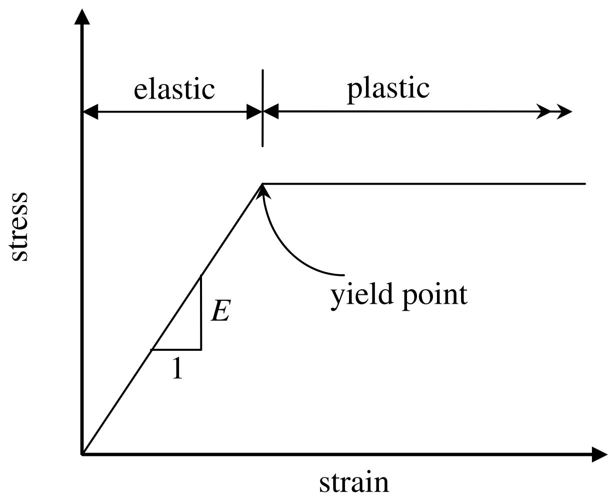

Fig. 1. Elastic-Perfectly Plastic Constitutive Relationship

laboratory testing and approximate ranges of the elastic parameters for various soils by Das B. M. (2004). [6]

Table 1. The parameters used in the elastic-plastic soil model.

Type of Soil	Elastic Modulus E, kN/m^2	Poisson's ratio, υ	Friction Angle ϕ^o	Cohesion c, kN/m^2
V. Dense Poorly Graded Sand with Silt and Gravel	55000	0.45	42	0
V. Dense Poorly Graded Sand with Silt	45000	0.45	40	0

6. Finite Element Stress Method

In addition to the limit equilibrium methods of analysis, SLOPE/W also provides an alternative method of analysis using the stress state obtained from SIGMA/W. These are GEO SLOPE program for static and dynamic finite element stress analyses respectively. The following sections outline the theoretical basis and the solution procedures used by the SLOPE/W Finite Element Stress method [13, 4, 7].

7. Stability Factor

The stability factor (S.F.) of a slope by the finite element stress method is defined as the ratio of the summation of the available resisting shear force along a slip surface ΣS_r to the summation of the mobilized shear force along a slip surface ΣS_m. In equation form, the stability factor (S.F.) is expressed as:

$$S.F. = \frac{\sum S_r}{\sum S_m} \tag{9}$$

The available resisting force of each slice is calculated by multiplying the shear strength of the soil at the base centre of the slice with the base length. Therefore, from the Mohr-Coulomb equation for a saturated soil; the available resisting force is: [13, 4]

$$S_r = S\beta = (C' + (\sigma_n - \mu_w)\tan\phi')\beta \tag{10}$$

where:

S = effective shear strength of the soil at the base centre of a slice

β = base length of a slice

σ_n = normal stress at base centre of a slice

Similarly, the mobilized shear force of each slice is calculated by multiplying the mobilized shear stress (τ_m) at the base centre of the slice with the base length.

$$S_m = \tau_m \beta \tag{11}$$

A local stability factor of a slice can also be obtained when the available resisting shear force of a slice is compared to the mobilized shear force of a slice [7,4,10].

$$Local\ S.F. = \frac{S_r}{S_m} = \frac{S\beta}{S\beta} \tag{12}$$

8. Normal Stress and Mobilised Shear Stress

To do stability analysis using the Finite Element Stress method, one needs to start by performing a finite element stress analysis (SIGMA/W). The information required from the stress analysis is the stress state as describe by σ_x, σ_y, and σ_{xy} at each Gauss point within each element. These stress values are used to compute the normal stress and the mobilized shear stress at the base centre of each slice [8].

9. Finite Element Computer Programme Used

A Geo-Slope program (SIGMA/W and SLOPE/W) was used in the finite element analysis carried out during this study. SIGMA/W program used to perform stress and deformation analysis of earth structure. The type of element considered in this work was the two dimensional quadrilateral element. The model which is considered in this work, is the elastic-plastic soil model.

SLOPE/W program used to compute the factor of safety of earth structure slope by finite element stress method and compared with the well established method of slices based on limit equilibrium.

The nodal points along the bottom boundary of the mesh were assumed to be fixed both horizontally and vertically. The nodes on the right and left ends of the mesh of the mesh were fixed in the horizontal direction while they were free to move in the vertical direction. All interior nodes were free to move horizontally and vertically.

10. The Survey and Laboratory Testing

The survey for Al-Furat river sides undergo by an engineering team. Depending on this survey, there was two sections taken (sections 1 and 2) to analysis them as shown in Figures 2 and 3.

In general, undisturbed samples of soils were taken for these sections for the laboratory testing. The soil samples were sent to the Soil Mechanics laboratory. The samples of each section were visually examined for initial classification before laboratory testing. The testing program included the following major tests on representative samples. These tests are:

a- Classification Tests

b- Engineering Tests on soil samples

All the tests were carried out in accordance with standards given in Table 2. The results of these tests are shown in Table 3.

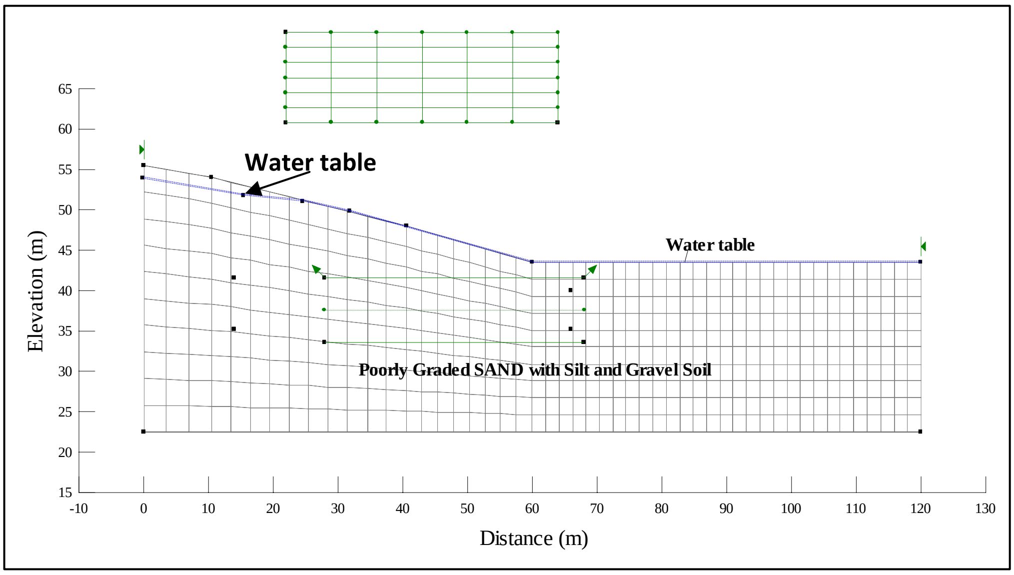

Fig. 2. The finite element mesh for section (1).

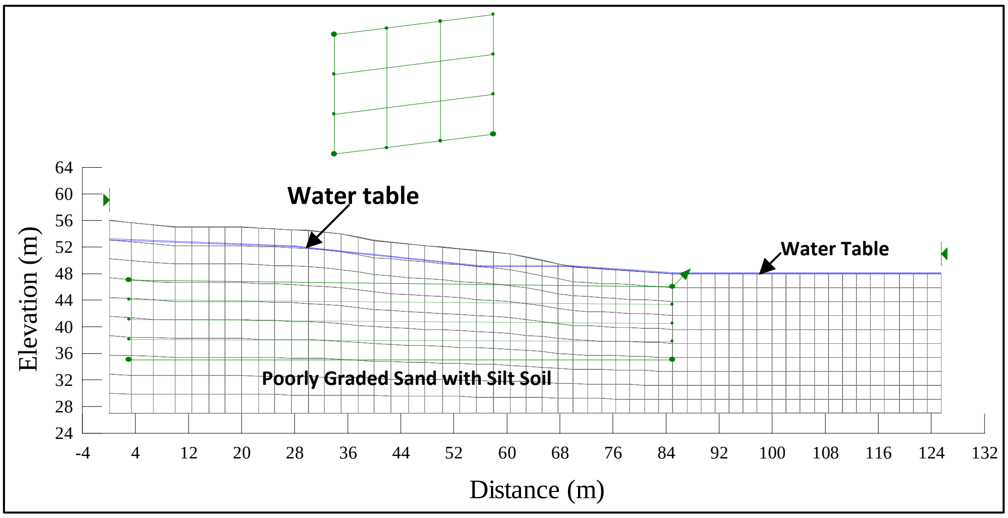

Fig. 3. The finite element mesh for section (2).

Table 2. Standards for laboratory testing.

Test	The Standard
Natural water content, (ω_n)	ASTM D 2216-71
Liquid Limit, (L.L)	ASTM D 423-66
Plastic Limit, (P.L)	ASTM D 424-59
Grain size analysis (sieve analysis and hydrometer analysis)	ASTM D 422-63
Undraind Direct Shear test	ASTM D 3080-72
Wet Unit weight (γ_t)	BS 1377-2

Table 3. Results of laboratory testing

Section No.	Moisture Content %	Liquid Limit %	Plastic Limit %	Grain Size Analysis			USCS System	Unit Weight	Strength Parameters	
				Finer %	Sand %	Gravel %		γ_t kN/m^3	c kN/m^2	ϕ°
1	8.9	NON	NON	5.25	63.88	30.87	SP-SM	16.9	0	42
2	20.7	NON	NON	5.4	91.45	3.15	SP-SM	18.4	0	40

11. Results of Finite Element Analysis

In this work, the attempt was to obtain laboratory measured parameters to simulate soil behaviour according to elastic-plastic soil model. The parameters as tabulated in Tables 1 and 3 were adopted in the two cases considered in the finite element analysis.

A-Section 1 (CASE I)

The results of finite element analysis of section 1, are shown in Figures 4 to 6. Section 1 is stable since the minimum factor of safety is 2.06 as shown in Fig. 4.

Contours of shear stresses are shown in Fig. 5. From this figure, it is clear that the maximum shear stress concentrates down of the abutment river due to soil mass movement downward.

Figure 6 shows the displacement vectors for section 1. From this figure, it can be seen that the displacement vectors reveal maximum values at top of river abutment and tend to concentrate near the centre of it.

Section 1 is found to be stable since:

- Minimum factor of safety = 2.06
- Maximum shear stress= 38.72 kPa
- Maximum vertical displacement = 58.8 mm
- Maximum horizontal displacement = 19.35 mm
  (at the mid-height of section 1)

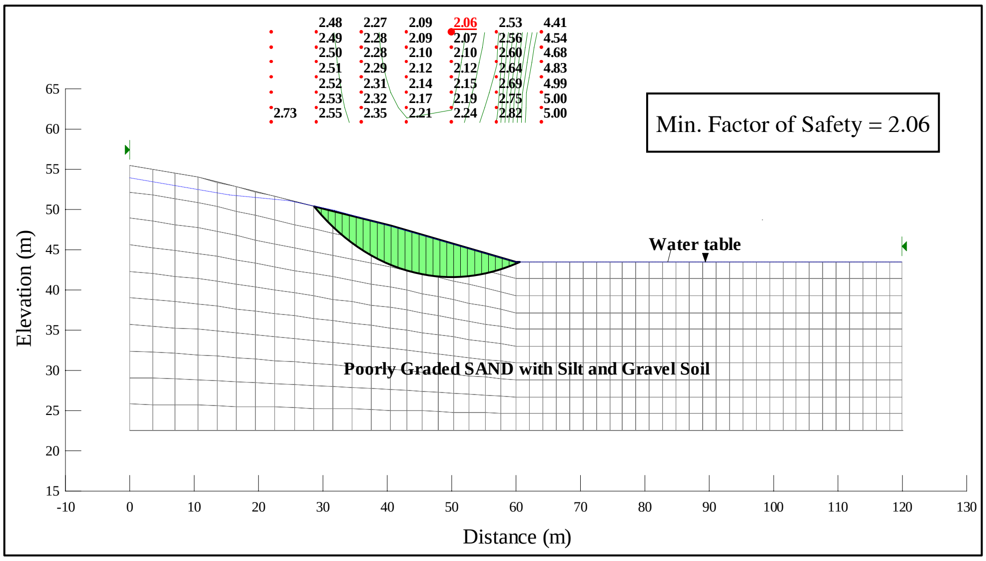

Fig. 4. The factor of safety of the section (1).

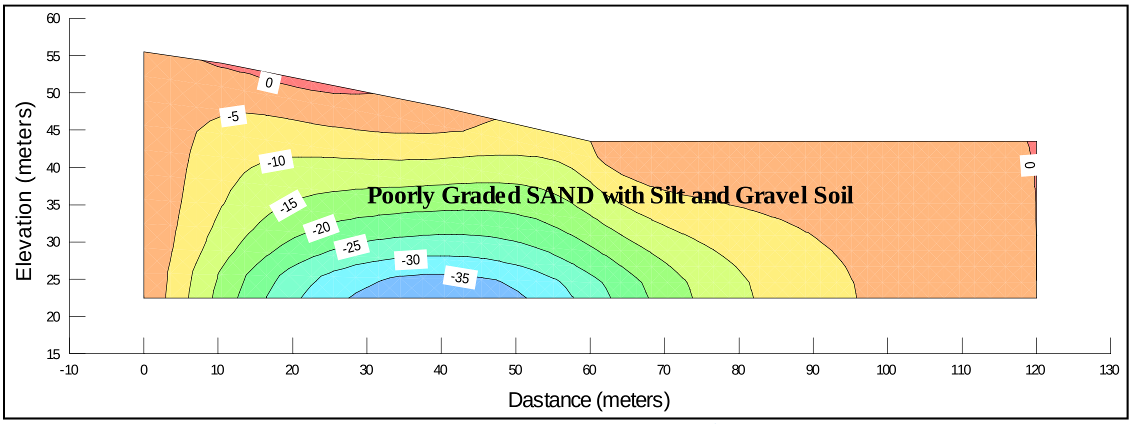

Fig. 5. Contours of shear stresses (kN/m^2) for section (1).

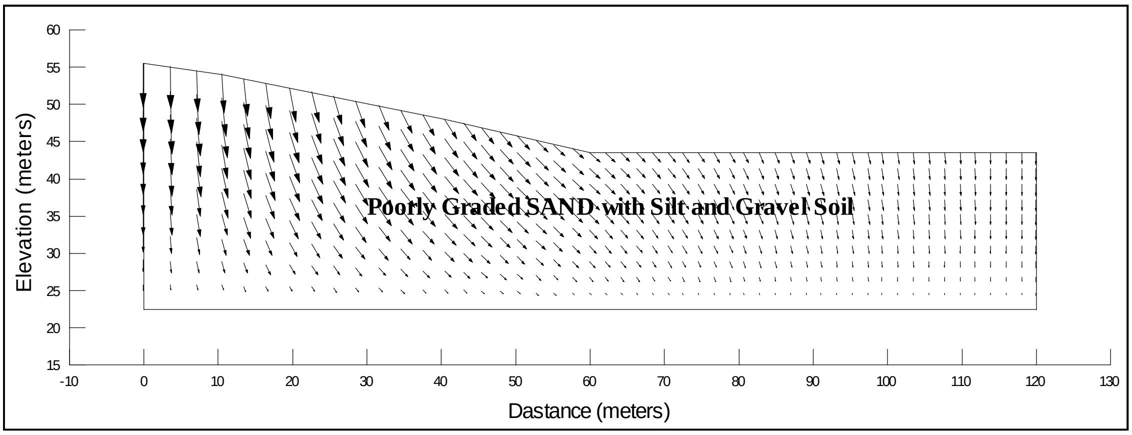

Fig. 6. Displacement vectors for section (1).

B- Section 2 (CASE II)

The results of finite element analysis are shown in Figures 7 to 9. The stability of this case, section 2, is well assured since the minimum factor of safety is 4.72 as shown in Fig. 7. Fig. 8 presents the deformed shape of the section 2 in case II, while the contours of Fig. 9 show shear stresses of section 2.

The section 2 is found to assure high degree of stability since:

- Minimum factor of safety = 4.72
- Maximum shear stress= 27.78 kPa
- Maximum vertical displacement = 52.7 mm (at the top of section 2)
- Maximum horizontal displacement = 17.16 mm (at the mid-height of section 2)

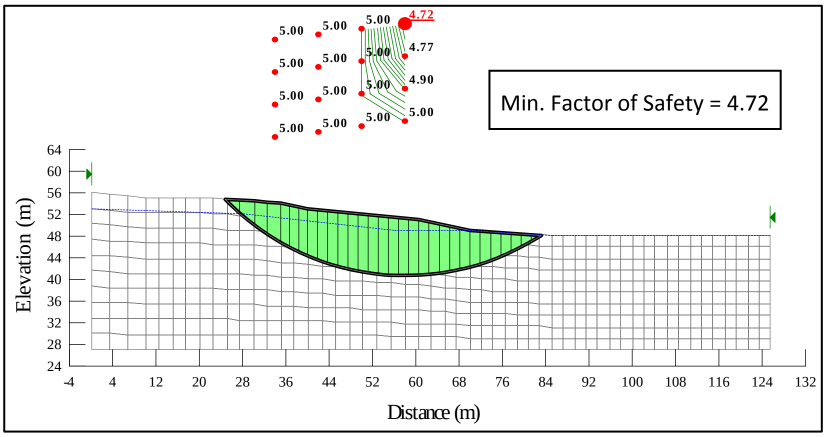

Fig. 7. The factor of safety of the section (2).

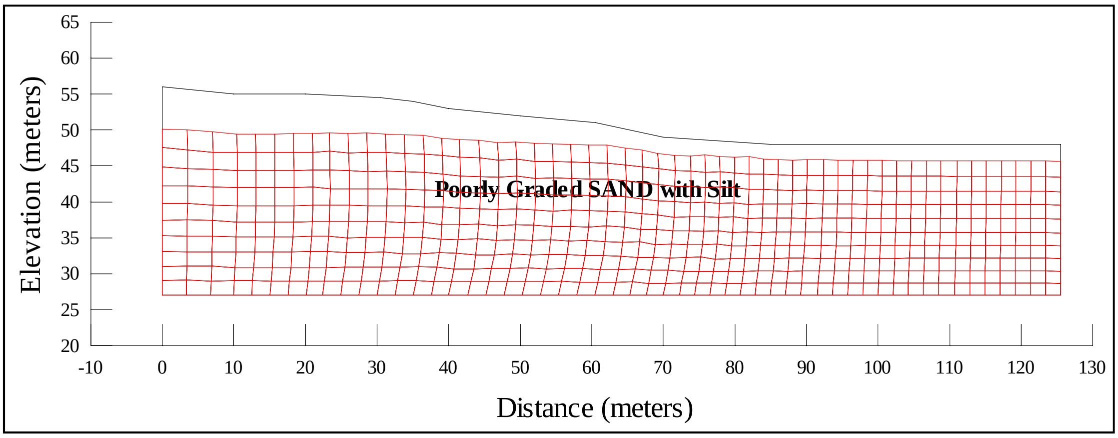

Fig. 8. Deformed shape of the section (2).

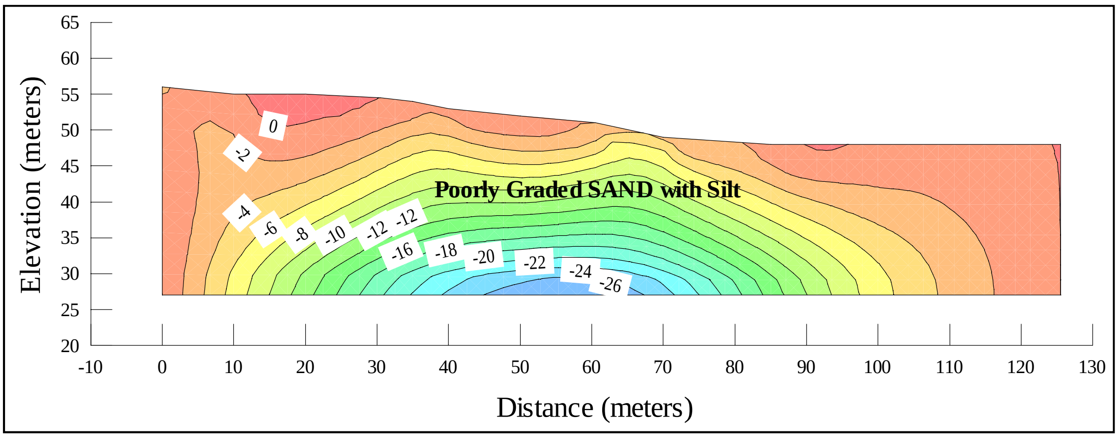

Fig. 9. Contours of shear stresses (kN/m^2) for section (2).

12. Comparison between the FEM the and Limit Equilibrium Methods

From Table 4, it can be seen that the minimum factor of safety for cases I and II which were found by the finite element method are more accurate than limit equilibrium methods because it intrinsically permits the realistic moulding of more aspects of problems than do alternative techniques. [5,2,10]

Table 4. Minimum factor of safety by FEM and limit equilibrium methods.

Type section	F.O.S. by F. E. Method	F.O.S. by Ordinary Method	F.O.S. by Bishop's Method of Slices	F.O.S. by Janbu Method of Slices	F.O.S. by Morgenstern-Price Method
Section 1 Case I	2.06	2.15	3.86	3.37	3.88
Section 2 Case II	4.72	4.20	5.20	4.78	5.20

13. Conclusions

The following conclusions are derived from the results presented:

1) The stability of abutment river sections is acceptable because of minimum factor of safety more than 1.5.
2) The finite element stability analysis method provides detailed information and an independent approach for determining the overall safety factor of the abutment river section base on either total and effective stress.
3) The case study presented above illustrates made to reduce the risk of failure.
4) The limit equilibrium and finite element methods give some different results , with the finite element method predictably yielding lower factors of safety.

References

[1] Adikari, G. S. N., Donald, I. B. and Parkin, A. K. (1982), "Analysis of the Construction Behaviour of Darmouth Dam", Proceedings of the Fourth International Conference on Numerical Methods in Geomechanics, Edmonton, Canada, Vol. 2.

[2] Bishop, A. W. and Morgenstern, N., 1960. Stability coefficients for earth slopes. *Geotechnique*, Vol. 10, No. 4, pp. 164 169.

[3] Clough, W., and Woodward, R.J. (1967), "Analysis of Embankment Stresses and Deformations", J. S. M. F. E., ASCE, Vol. 93, SM4.

[4] Christian, J. T., Ladd, C.C. and Baecher, G. B., 1994. Reliability Applied to Slope Stability Analysis. *Journal of Geotechnical Engineering*, Vol. 120, No. 12. Pp. 2180-2207.

[5] Desai, C. S. and Abel, T. F. (1972), "*Introduction to the Finite Element Method*" Van Nostrand Reinhold, London.

[6] Das, B. M. (2004), "*Principles of Foundation engineering*" Brooks/Cole-Thomson Learning.

[7] Fredlund, D. G., and Krahn, J., 1977. Comparison of slope stability methods of analysis. *Canadian Geotechnical Journal*, Vol. 14, No. 3, pp. 429 439.

[8] Fredlund, D. G., 1974. Slope Stability Analysis. User's Manual CD-4, Department of Civil Engineering, University of Saskatchewan, Saskatoon, Canada.

[9] Kulhawy, F. and Duncan, J. (1972), "Stresses and Movements in Oroville Dam", J. S. M. F. D., ASCE, Vol. 98, SM7.

[10] Krahn, J., Price, V. E., and Morgenstern, N. R., 1971. Slope Stability Computer Program for Morgenstern-Price Method of Analysis. User's Manual No. 14, University of Alberta, Edmonton, Canada.

[11] Nagtegaal J. C., Parks D. M. and Rice J. R., "On numerically accurate finite element solutions in the fully plastic range", *Comp. Meth. Appl. Mech. and Eng.* 4, 153-177 (1974).

[12] Veiga Pinto, A. and Neves, E. M. (1985), “Prediction of Beliche Dam Behaviour during Reservoir Filling”, Proceeding of The Eleventh International Conference on Soil Mechanics and Foundation Engineering, Vol. 4.

[13] Whitman, R. V. and Bailey, W. A., 1967. Use of Computer for Slope Stability Analysis. *Journal of the Soil Mechanics and Foundation Division of ASCE*, Vol. 93, No. SM4.