

Development of Computer Program to determine Runway Length Required for Airport Design

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الخلاصة

ان اختيار طول المدرج هو واحد من اهم القرارات لاي مصمم مطارات. الطول المطلوب لاحتواء الطائرات التي تستخدم المطار هو العامل الاساسي للتصميم. هذا البحث يصف الدور المهم الذي يلعبه طول المدرج لاي تصميم مقترح للمطارات. تم تطوير برنامج سمي (RUNWLD) لهذا الغرض خلال فترة البحث لايجاد طول المدرج معتمدا على طريقة (FAA). البرنامج يتنبأ بالطول لخيط واسع من الطائرات. البرنامج مكتوب بلغة (visual basic). البيانات التي ادخلت في البرنامج اخذت من الجداول والاشكال والمواصفات التي تستخدمها (FAA). البرنامج سهل الاستعمال ويوفر الوقت باستخراج النتائج مقارنة بالطريقة التقليدية التي تستخدمها (FAA).

Abstract

Selecting a design runway length is one of the most important decisions an airport designer makes. The length required to accommodate the most demanding airplanes anticipated to use an airport is a fundamental airfield design factor. This paper describes the important role which the runway length is playing in any proposed airport to be designed.

Computer program named (RUNWLD) was developed during this research period to determine the runway length depending on the Federal Aviation Administration (FAA) methodology. (RUNWLD) predicts the planned and basic runway lengths for various mix of airplanes anticipated to use a proposed airports.

The program was written in visual basic programming language. The data used in this program is concluded from the charts, tables, and circular advisory adopted by (FAA) methodology. The developed program (RUNWLD) is easy tool and user friend, in addition to that it save time while getting results comparing to the traditional (FAA) method.

Keywords: runway length, airport, airplane, FAA, program

Introduction

Air transportation provides the backbone for passenger transport over moderate to long distances in the world, and it is becoming an increasingly important mode for short-range travel and cargo transport as well. As a consequence, there is a growing demand for use of available airspace, a heightened concern for safety, and a greater likelihood that poor weather will be encountered during typical flight operations.

Air transportation affects this nation's competitiveness in two ways: 1) It is an important component of the industrial base, having made a positive contribution to balance of payments for many years, and 2) it is an enabling technology for all other industries, providing a major avenue for commerce by moving both people and cargo. Continued improvements in safety and efficiency of civil aviation are urgently needed to preserve not only national competitiveness but the quality of our lives and our environment in the process.

New technologies hold promise for increasing the productivity, reliability, and safety of the air transportation system, but they introduce uncertainty and present new challenges for certification. It is necessary, therefore, to create new ways of dealing with these problems and, in the process, to look after a new generation of researchers capable of solving problems yet to come.

Selecting a design runway length is one of the most important decisions an airport designer makes. The length required to accommodate the most demanding airplanes anticipated to use an airport is a fundamental airfield design factor.

The runway length determines the size, cost of the airport, and controls the type of aircraft it will serve. It may limit the payload of the critical aircraft and the length of journey it can fly. Runway must be long enough for safe landings and takeoffs, it accommodate differences in pilot skill, variety of aircraft types, and operational requirements.

Airfield Requirements

Airfield facilities are those that are related to the arrival, departure, and ground movement of aircraft. Airfield facility requirements are addressed for the following areas (1):

- 1) Airfield Capacity.
- 2) Airfield Design Standards.
- 3) Runway Orientation, Length, Width, and Pavement Strength.
- 4) Taxiways.
- 5) Airport Visual Aids.

- 6) Airport Lighting.
- 7) Radio Navigational Aids & Instrument Approach Procedures.
- 8) Helicopter Facilities.
- 9) Other Airfield Recommendations.

The runway configuration and its features represent the important factors in the airfield requirements. The runway length required to accommodate the most demanding airplanes anticipated to use an airport is a fundamental airfield design factor.

Factors governs Runway Length Suitability

Various factors, in turn, govern the suitability of those available runway lengths, most notably airport elevation above mean sea level, temperature, wind velocity, airplane operating weights, takeoff and landing flap settings, runway surface condition (dry or wet), effective runway gradient, presence of obstructions in the vicinity of the airport, and, if any, locally imposed noise abatement restrictions or other prohibitions. Of these factors, certain ones have an operational impact on available runway lengths. That is, for a given runway the usable length made available by the airport authority may not be entirely suitable for all types of airplane operations (2).

The goal is to construct an available runway length for new runways or extensions to existing runways that is suitable for the forecasted critical design airplanes.

Modeling

The formulation of a system model is the most important step towards the solution of scientific problems. A model permits the designer to predict, with some degree of certainty, the behavior of the system under various conditions. The fundamental conceptual device is an image of reality portraying the system and the interaction between the components (3).

Models are extensively used as aids in the description and analysis of problems, test the behavior of a new system or operating procedure prior to its actual construction, and the need to test alternate system under identical conditions.

(FAA) Methodology to determine recommended Runway Length.

As specified in Federal Aviation Administration (FAA) planning criteria, the recommended length for a primary runway must be determined by considering either the family of aircraft having similar performance characteristics or a specific aircraft requiring the longest runway. In either case, the choice should be based on aircraft that are anticipated to use the runway on a regular basis, which is defined by the FAA Advisory Circular (AC).

Runway length requirements were estimated using procedures outlined in FAA Advisory Circular (AC) 150/5325-4B, Runway Length Requirements for Airport Design, along with additional information provided in aircraft data charts from aircraft manufacturers. The runway length analysis methodology contained in AC 150/5325-4B considers both arrivals and departures; however, departures typically require longer runway lengths (2).

This Advisory Circular (AC) provides guidelines for airport designers and planners to determine recommended runway lengths for new runways or extensions to existing runways. The Advisory Circular stated an assumptions, definitions, and procedure for determining the recommended runway length.

a. Assumptions and Definitions.

(1) Design Assumptions. The assumptions used by this AC are approaches and departures with no obstructions, zero wind, dry runway surfaces, and zero effective runway gradient.

(2) Critical Design Airplanes. The listing of airplanes (or a single airplane) that results in the longest recommended runway length. The listed airplanes will be evaluated either individually or as a single family grouping to obtain a recommended runway length.

(3) Small Airplane. An airplane of 12,500 pounds (5,670 kg) or less maximum certificated takeoff weight.

(4) Large Airplane. An airplane of more than 12,500 pounds (5,670 kg) maximum certificated takeoff weight.

(5) Maximum Certificated Takeoff Weight (MTOW). The maximum certificated weight for the airplane at takeoff, i.e., the airplane's weight at the start of the takeoff run.

(6) Regional Jets. Although there is no regulatory definition for a regional jet (RJ), an RJ for this advisory circular is a commercial jet airplane that carries fewer than 100 passengers.

(7) Crosswind Runway. An additional runway built to compensate primary runways that provide less than the recommended 95 percent wind coverage for the airplanes forecasted to use the airport.

(8) Substantial Use Threshold. Federally funded projects require that critical design airplanes have at least 500 or more annual itinerant operations at the airport (landings and takeoffs are considered as separate operations) for an individual airplane or a family grouping of airplanes. Under unusual circumstances, adjustments may be made to

the 500 total annual itinerant operations threshold after considering the circumstances of a particular airport. Two examples are airports with demonstrated seasonal traffic variations, or airports situated in isolated or remote areas that have special needs.

(9) Itinerant Operation. Takeoff or landing operations of airplanes going from one airport to another airport that involves a trip of at least 20 miles. Local operations are excluded.

(10) Effective Runway Gradient. Is the difference between the highest and lowest elevations of the runway centerline divided by the runway length.

b. Procedure for Determining Recommended Runway Lengths.

AC 150/5325-4B uses a five-step process to determine recommended runway lengths for a selected list of critical design airplanes. Generally, the five steps are as follows:

Step -1: Identify the critical design airplanes that will make regular use of the proposed runway for an established planning period of at least five years.

Step -2: Identify the airplanes that will require the longest runway lengths at maximum certificated takeoff weight (MTOW). The second step in determining a recommended runway length through the standard FAA process is to break down the potential range of critical design airplanes identified in Step -1 into relevant weight groupings or categories. The purpose of this effort is to narrow down the full range of potential design aircraft and focus the analysis on those most critical to runway length. Note that this grouping process is based on the individual aircraft's maximum certified takeoff weight. AC 150/5325-4B groups aircraft into three categories:

- 1) MTOW of 12,500 pounds or less.
- 2) MTOW over 12,500 pounds, but less than 60,000 pounds.
- 3) MTOW 60,000 pounds or more or Regional Jets.2.

Step -3: Determine the method that will be used for establishing the recommended runway length. The standard FAA process is to establish the method to be utilized to analyze the 60,000 pound or more weight category identified in Step -2. Note that AC 150/5325-4B acknowledges the potential wide variety of operational requirements contained in that broad category. Therefore, it allows for the analysis of individual aircraft, as opposed to the broad, family groupings of aircraft that it recommends for smaller aircraft.

AC 150/5325-4B provides the following options for obtaining data for aircraft of more than 60,000 pounds:

- Analyzing performance charts published by the airplane manufacturers.
- Contacting the airplane manufacturer for specific information; or
- Contacting air carriers for their specific operational requirements.

Step -4: Select the recommended runway length from among the various runway lengths generated by Step -3.

Step -5: Apply any necessary adjustment to the obtained runway length, when instructed by the applicable chapter of this AC, to the runway length generated by step -4 to obtain a final recommended runway length. For instance, an adjustment to the length may be necessary for runways with non-zero effective gradients.

To complete the picture for step -5, it is essential to explain the basic runway length concept.

Basic Runway Length

It is the most important airside design feature and should be linked to other physical characteristics of the airport. To provide a meaningful relationship between runway length and other physical characteristics of the airside, the actual runway length must be converted to standard sea level conditions by removing the local effects of elevation, airport reference temperature, and gradient. Then the resulting length is called the basic runway length (4).

$$\text{Basic runway length} = \text{planned runway length} / F_e * F_t * F_g$$

Where:

F_e is the elevation factor = $0.07E+1$, where, E is the airport elevation in 1000ft.

F_t is the temperature factor = $0.01 [T-(15-1.956E) +1]$, where,

$T = T_1 + (T_2-T_1)/3$, where,

T is the airport reference temperature

T_1 is monthly mean of mean daily temperature for hottest month.

T_2 is the monthly mean of maximum daily temperature for same month.

F_g is the gradient factor = $0.10G + 1$, where, G is the effective runway gradient which is defined hereinabove.

Computer Program Formulation and Modeling

Program formulation

The computer program developed during this research period is called **RUNWLD** (**RUNWay Length Determination**). It was written using visual basic programming language. The model is of deterministic type and calculates the recommended runway length for airports according to Federal Aviation Administration (FAA) methodology for various aircraft types and sizes. The aircraft maximum takeoff weight plays the main role in the determination of the runway length.

The program permits measurement of a full range of air traffic characteristics and is allowing many alternative designs to be tested.

The program permits also the necessary adjustments belong to the airport elevation, runway gradient, and airfield temperature.

The computer program is user friend and was designed in modular manner and a great deal of care was made to make allowances for future developments.

Typical solved example showing the input and output stage interfaces for the developed model is presented in end of this research.

Computer Program Modeling

The computer program modeling or development was achieved by three main stages in addition to the modeling of the interferences of input and output data.

The first stage of the computer program modeling is the classification of the various type of aircrafts anticipated to use the planned runway length according to their weights. This stage was implemented through three steps as described below:

The first step in the computer program modeling is to identify the critical design airplanes that will make regular use of the proposed runway for an established planning period of at least five years.

The second step is to establish a table contains the weight for each airplane according to its manufacturer, type, and series as shown in Table (1) below. The runway length for each airplane also is listed in this table depending on the FAA specifications.

The third step is to classify these design airplanes into three categories according to their weights. These limitations were made according to FAA concepts (AC 150/5325-4B). Table (2) represents the three groups. The classification is as follows:

- 1) airplanes with maximum takeoff weight of 12,500 pounds or less.
- 2) airplanes with maximum takeoff weight Over 12,500 pounds, but less than 60,000 pounds.
- 3) airplanes with maximum takeoff weight of 60,000 pounds or more or Regional Jets.2.

The second stage of program development is to convert the chart which is adopted by Federal Aviation Administration (FAA) into numerical values contains the magnitude of each runway length corresponding to the airplane weights and added to table (1). These values in this table will be adopted in the computer program to calculate the planning runway lengths for various mix of airplanes.

The third stage is focusing on the planning runway length adjustments. These adjustments result in determining the basic runway length. The basic runway length represents the recommended runway length adopted in an airport design. The adopted equation used in this program is as follow:

$$\text{Basic runway length} = \text{planned runway length} / F_e * F_t * F_g$$

The definitions of the equation is found hereinabove.

By the end of this stage, the program is terminated and the intended result is obtained. The design of the airport is mainly depends on this value.

Table (1):-Runway length according to aircraft features.

Aircraft			Maximum takeoff weight, lb (MTOW)	Takeoff Runway length, ft
Manufacturer	Type	Series		
Boeing	737	800		9700
Boeing	767	200ER		9200
Boeing	757	300	136500	10400
Boeing	757	200	240000	10300
Boeing	767	300	350000	8900
Boeing	727 (JTSD-7)	200	167000	12200
Boeing	727 (JT8D-7)	200	189000	12900
Boeing	727 (JT8D-7)	100	167000	12300
Boeing	737 (CFM56-3-BI)	300	124500	7200
Boeing	747 (JT9D-7A)	200B	736000	12200
Boeing	757 (RBII-535E4)	200	240000	7100
Boeing	767 (CF6-80A)	200	300000	6700
Boeing	767 (JT9D-7R4D)	200ER	351000	9100
Boeing	747 (PW4256)	400	496000	11200
Boeing	747 (JTgD-7A)	200B	785000	14400
DC	9 (JTSD)	30	100000	9500
DC	10 (CF6-6D)	10	400000	12700
DC	10 (CF6-50C)	30	555000	15300
MD	(JTSD-217)	82	149500	9400
L	I011 (RB211-22B)	385-I	403000	10400
Airbus	(CFM56-5A1)	320		10100
Airbus	319	100		9200
Airbus	321	200		8900
Airbus	319	100s		8800
Canadair	(CF34-3B1)	200LR		6900
Canadair	(CF34-8C1)	700ER		5600
Canadair	(CF34-8C5)	900		6800
Embraer		145		6900
Embraer	Brasilia (PW118)	120		5400
Learjet	Business jet	30		5550

Reference:- FAA concepts (AC 150/5325-4B).

Table (2):- Airplane classification according to their weights.

Small airplanes with less than 10 passenger seats	Runway Length,ft
To accommodate 75 percent of these small airplanes	2,480 feet
To accommodate 95 percent of these small airplanes	3,030 feet
To accommodate 100 percent of these small airplanes	3,600 feet
Small airplanes with 10 or more passenger seats	3,600 feet

Reference: FAA concepts (AC 150/5325-4B).

Model Validation

To test the validity of the developed model outputs, it is necessary to solve the same example indicated in this research as a typical model input and output interferences by traditional (FAA) method using the values listed in tables (1), and (2) which they were drawn from the charts adopted by Advisory Circular (AC 150/5325-4B). The results were then compared to see if there is a significant difference between the two results or not.

From table (1), to determine the basic takeoff runway length for Boeing 767 aircraft of type (JT9D-7R4D) with series 200ER having maximum takeoff weight, 351000 lb is 9100 ft, which is identical to the value appeared in the computer model output. This comparison led to the fact that the computer model is working properly and is valid to use as a user friend tool for computing the runway length necessary for airport design.

Discussion and Recommendation

The developed computer program (**RUNWLD**) predicts the planned and basic runway lengths for various mix of airplanes anticipated to use a proposed airports.


The developed program adopts the Federal Aviation Administration (FAA) methodology. This method uses charts and tables to complete the determination of the runway length. These charts and tables are not available every where and in any time, therefore the developed program (**RUNWLD**) is easy tool and user friend, in addition to that it save time while getting results comparing to the traditional (FAA) method. (**RUNWLD**) may run on any available computer because of its small size and no need of high technology computers.



Development of this computer program is necessary to model other important design factors for any proposed planned airport, for example, the design of taxiways, aprons, and terminals. Recommendation required for developing and extending the validity of the model, considering a wide range of model applications is also necessary.

It is recommended for future development to simulate more types and sizes of aircraft rather than collected in this research tables.

References

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General	Input Data	Correction Var.	Out Put	About
Project Name (Airport Name): <input type="text" value="City International Airport"/>				
Analyst: <input type="text" value="Trans Company"/>		Date: <input type="text" value="03/5/2008"/>		
				

General	Input Data	Correction Var.	Out Put	About
<div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> <p>MTOW less than 12500 lb less than 10 passenger seats</p> <p><input type="radio"/> 75%</p> <p><input type="radio"/> 95%</p> <p><input type="radio"/> 100%</p> <p><input type="radio"/> 10 or more</p> <p style="text-align: right;"><input type="button" value="Done"/></p> </div> <div style="width: 45%;"> <p>MTOW more than 160000 lb</p> <p>Manufacturer: <input type="text" value="boeing"/> *</p> <p>Type: <input type="text" value="767 (j9d-7r4d)"/></p> <p>Series: <input type="text" value="200er"/></p> <p>MTOW: <input type="text" value="351000"/></p> <p style="text-align: right;"><input type="button" value="Done"/></p> </div> </div>				
<div style="display: flex; justify-content: space-around;">   </div>				

General	Input Data	Correction Var.	Out Put	About
<div style="border: 1px solid gray; padding: 10px; width: 80%; margin: auto;"> <p>Differ. in Runway Level: <input type="text" value="50"/></p> <p>Airport Level(E): <input type="text" value="0"/> (in 1000 ft.) *</p> <p>Air Temp. (T1): <input type="text" value="35"/></p> <p>Air Temp. (T2): <input type="text" value="45"/></p> <p style="text-align: right;"><input type="button" value="Done"/></p> </div>				



Prediction of Ultimate Bearing Capacity of Shallow Foundations on Cohesionless Soils Using Back Propagation Neural Networks (BPNN)

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الخلاصة

يهدف هذا البحث الى استكشاف امكانية استخدام الشبكات العصبية بتقنية الانتشار الرجعي للخطأ لايجاد قابلية التحمل القصوى للاسس الضحلة المقامة على الترب غير المتماسكة. قاعدة بيانات شملت ما مجموعه 97 حالة لفحوص تحميل على اسس بابعاد حقيقية واخرى لنماذج صغيرة ، لبناء وااثبات نماذج الشبكات العصبية. العوامل الخمسة التالية يمكن اعتبارها من العوامل ذات التأثير الأكبر على قابلية التحمل القصوى وقد اعتبرت كمعطيات للنموذج وتشمل عرض الاساس، عمق الطمر للاساس، نسبة الطول الى عرض الاساس، الكثافة الجافة او المغمورة وزاوية الاحتكاك الداخلي للتربة ، في حين إن قابلية التحمل القصوى هي نتيجة النموذج. تم اجراء تقييم شامل للنموذج باستخدام معامل الارتباط، جذر معدل مربع الخطأ، و معدل الخطأ المطلق. وقد اظهرت النتائج فاعلية النموذج في ايجاد قابلية التحمل القصوى. تم مقارنة النموذج مع اكثر المعادلات المستخدمة شيوعاً لايجاد قابلية التحمل القصوى وقد اظهرت النتائج افضلية النموذج على تلك النظريات.

Abstract:

This study explores the potential of back propagation neural networks (BPNN) computing paradigm to predict the ultimate bearing capacity of shallow foundations on cohesionless soils. The data from 97 load tests on footings (with sizes corresponding to those of real footings and smaller sized model footings) were used to train and validate the model. Five parameters are considered to have the most significant impact on the magnitude of ultimate bearing capacity of shallow foundations on cohesionless soil and are thus used as the model inputs. These include the width of the footing, depth of embedment, length to width ratio, dry or submerge unit weight and angle of internal friction of the soil. The model output is the ultimate bearing capacity. Performance of the model was comprehensively evaluated. The values of the performance evaluation measures such as coefficient of correlation, root mean square error, mean absolute error reveal that the model can be effectively used for the bearing capacity prediction. BPNN model is compared with the values predicted by most commonly used bearing capacity theories. The results indicate that the model perform better than the theoretical methods.

KEYWORDS: Ultimate bearing capacity; Shallow foundations; cohesionless soil; back propagation neural network (BPNN); prediction

1. Introduction:

Every foundation design requires satisfying two major criteria: ultimate bearing capacity and limiting settlement of foundations [1]. Of these two criteria, the ultimate bearing capacity is governed by shearing strength of the soil and is estimated by the theories of Terzaghi [2], Meyerhof [3], Vesic [4] and others. The basis for most of the bearing capacity theories is the limit equilibrium method. The bearing capacities thus obtained are validated through laboratory studies by numerous researchers. However, the experimental researches are generally carried out on smaller sized models, which are highly scaled down models compared to real footings. Consequently, many researchers (e.g., [5–8]) have cautioned that one should be very careful when extrapolating findings of experiments conducted on small footings that have a width of a few inches, to the large sized footings. The reason for this is attributed to the increase in shearing strain along the slip line with the increase in width of the foundation and the ratio of mean grain size of the soil and the footing width [9]. The scale effect due to particle size becomes insignificant when the ratio of mean grain size and the width of footing is less than a certain limit, depending on the type of the soil [10]. For large-scale foundations on dense sand, shearing strains show considerable variation along the slip line and the average mobilized angle of shearing resistance along the slip line is smaller than the maximum value of the angle of shearing resistance (ϕ_{pmax}) obtained by plane strain shear tests. Therefore, the bearing capacity formula generally over estimates the bearing capacities of actual foundations on dense sand, if ϕ_{pmax} is used [11]. Hence, an alternative method is required that provides better estimates of bearing capacity.

During the last two decades several researchers have developed effective modeling tools using Neural Networks (NNs) approach. NNs have been applied to many geotechnical engineering problems, including the prediction of the bearing capacity of piles, settlement predictions, liquefaction and slope stability [12]. This indicates that NNs can be used for both prediction and forecasting of events. The major advantage of NNs is that they can be updated easily as and when new data become available that eliminates the need for a specialist to reanalyze the old and new data, update the old design aids or equations and/or propose new equations [13].

This paper demonstrates the applicability back propagation neural network algorithm, in developing an effective model for predicting the ultimate bearing capacity of shallow foundations on cohesionless soils, and to undertake a comparative study with the commonly used bearing capacity theories. The database, which consists of load test results of large-scale footings and smaller sized model footings, is used to develop and verify the model. The performance of the model is then compared with the most commonly used bearing capacity theories.

2. NEURAL NETWORKS (NN)

NN is a computational tool, which attempts to simulate the architecture and internal operational features of the human brain and nervous system [14]. Three or more layers, which includes an input layer, an output layer and a number of hidden layers in which neurons are connected to each other with modifiable weighted interconnections (Fig. 1), form NN architectures. Each neuron has an associated transfer function, which describes how the weighted sum of its inputs is converted to the results into an output value. Each hidden or output neuron receives a number of weighted input signals from each of the units of the preceding layer and generates only one

output value (Fig. 2). This NN architecture is commonly referred to as a fully interconnected feed-forward multi-layer perceptron. In addition, there is also a bias, which is only connected to neurons in the hidden and output layers with modifiable weighted connections. The number of neurons in each layer may vary depending on the problem.

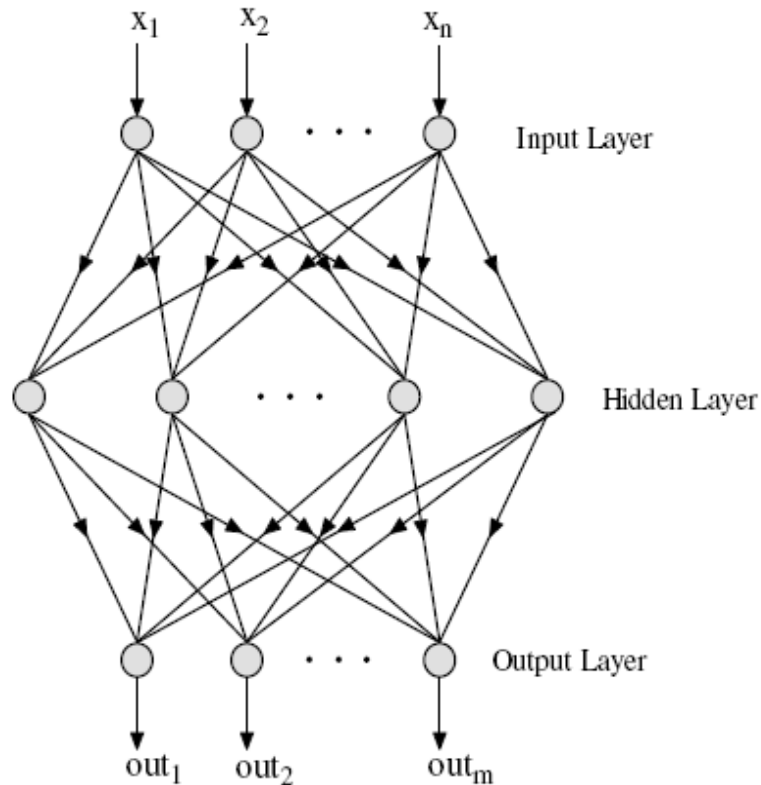


Fig. 1. A typical MLP neural network.

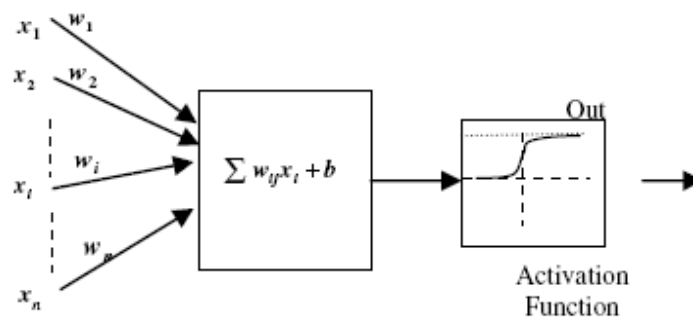


Fig. 2. The structure of an artificial neuron.

The most widely used training algorithm for multilayered feed-forward networks is perhaps the back-propagation (BP) algorithm [see, for instance, [15, 16]]. The BP algorithm basically involves two phases. One is the forward phase where the activations are propagated from the input to the output layer. The second is the backward phase where the error between the observed

actual value and the desired nominal value in the output layer is propagated backwards in order to modify the weights and bias values. Before training a feed work network, the inputs and the outputs of training and testing sets must be initialized. In the forward phase, the weighted sum of input components is calculated as

$$net_j = \sum_{i=1}^n w_{ij} x_i + bias_j \quad (1)$$

where net_j is the weighted sum of the j th neuron for the input received from the preceding layer with n neurons, w_{ij} is the weight between the j th neuron and the i th neuron in the preceding layer, x_i is the output of the i th neuron in the preceding layer. The output of the j th neuron out_j is calculated with a sigmoid function as follows:

$$out_j = f(net_j) = \frac{1}{1 + e^{-(net_j)}} \quad (2)$$

The training of the network is achieved by adjusting the weights and is carried out through a large number of training sets and training cycles. The goal of the training procedure is to find the optimal set of weights, which would produce the right output for any input in the ideal case. Training the weights of the network is iteratively adjusted to capture the relationship between the input and output patterns. The training of the network is accomplished by adjusting the weights and is carried out through a large number of training sets and training iterations. The goal of the learning procedure is to find the optimal set of weights, which in the ideal case would produce the right output for any input. The output of the network is compared with a desired response to produce an error. The performance of the MLP is measured in terms of a desired signal and the criterion for convergence.

In this study, a computer program has been developed and performed under EXCEL worksheet. The back-propagation learning algorithm has been used in feed-forward with one hidden-layer, back propagation algorithm (BP), as one of the most famous training algorithms for the multi-layer perceptron (MLP), is a gradient descent technique to minimize the error E for a particular training pattern, For adjusting the weight (w_{ij}) from the i th input unit to the j th output, in the batched mode variant the descent is based on the gradient

$$\tilde{\Delta} E_{ij} = -\frac{\partial E}{\partial w_{ij}} \quad (3)$$

for total training set

$$Dw_{ij}(n) = -\epsilon \frac{\partial E}{\partial w_{ij}} + \alpha Dw_{ij}(n-1) \quad (4)$$

The gradient gives the direction of error E , The parameters ϵ and α are the learning rate and momentum term, respectively.

3. DEVELOPMENT OF MODEL FOR ULTIMATE BEARING CAPACITY

One of the more important steps in the model development for the estimation of bearing capacity of shallow foundations is identification of parameters that affect the bearing capacity, for which some of the traditional bearing capacity methods [2–4, 17] are examined. Despite the fact that the bearing capacity values obtained through these methods differ considerably, the basic form of equation is the same for all the methods, which is as follows for foundations in cohesionless soil:

$$q_u = gDN_q S_q d_q + 0.5BgN_g S_g d_g \quad (5)$$

where B – width of foundation, D – depth of foundation, γ – unit weight of sand (below and above the foundation level), N_q , N_γ – bearing capacity factors, S_q , S_γ – shape factors and d_q , d_γ – depth factors. Though various researchers propose different equations for the computation of these factors they primarily depend on the angle of shearing resistance of the sand and the geometry of the foundation. It is clear from the above that the bearing capacity of foundation depends on a considerable number of physical parameters of the foundation and the soil in which the foundation is embedded. Among the parameters related to the foundation, the main factors affecting the bearing capacity are its width (least lateral dimension, B), length of footing (L), shape (square, rectangular and circular) and depth of embedment (D). The depth of foundation has the greatest effect on the bearing capacity of all the physical properties of the foundation [6]. The main parameters in regard to the soil (sand)

are its angle of shearing resistance and the unit weights from above and below the water table, if present. There are some other factors such as compressibility and thickness of the soil layer beneath the foundation that contribute to a lesser degree. Of all the properties of a soil, the angle of shearing resistance, ϕ , has greatest influence on the bearing capacity, which increases with the relative density of the soil. The bearing capacity is directly proportional to the unit weight of the soil and is influenced by the location of water table.

The effect of compressibility is small, except for loose densities, and is generally less important in bearing capacity computation [6]. Moreover, there are insufficient data to consider compressibility as well as thickness of soil stratum. Therefore they are not considered explicitly in this study. Further, the recent study by Foye et al. [18], based on reliability analysis of the design of foundations identified B, L, D, γ and ϕ as the important parameters that affect bearing capacity, and also discussed the degree of influence of these parameters on N_γ , S_γ , d_γ . Based on the above, the five input parameters used for the model development in this study are width of footing (B), depth of footing (D), footing geometry (L/B), unit weight of sand (γ) and angle of shearing resistance (ϕ). Ultimate bearing capacity (q_u) is the single output variable.

n The data used for model development

The data used for calibrating and validating the model were collected from literature, which include load test data on real sized foundations, as well as the corresponding information regarding the footing and soil. The data base thus developed comprises a total of 97 data sets, which consists of results of square, rectangular and strip footings of different sizes tested in sand beds of various densities. To enhance the performance of the model, the data used are more evenly distributed (i.e., the number of data for large sized footings and smaller sized models are equal). Of the 97 data sets, 47 are from load tests on large-scale footings and 50 are from smaller sized model footings. Of the 47 large-scale footing data, 24 were reported by Muhs and WeiB [19], 11 by WeiB [20], 5 by Muhs et al. [21], 2 by Muhs and WeiB [22], 5 by Briaud and Gibbens [23]. The experimental results of smaller scale model footings were reported by Gandhi

[24]. The data used are presented in Table 1. The large-scale tests at the test area of DEGEBO, Berlin were conducted in a submerged condition and hence submerged unit weights are used for these tests. The angle of shearing resistance as reported by the respective authors of the paper are adopted in the analysis, despite the mobilized angle of shearing resistance at failure for the axisymmetric and plain strain conditions are different. However, the difference in the angle of shearing resistance between these two conditions is not more than 10%. Moreover, in the case of laboratory model tests the angles of shearing resistance used are obtained from the direct shear tests conducted at very low normal stresses. Thus, the effect of dilation is also included. In the case of large-scale footings the ultimate load is defined as the load corresponding to the point where the slope of the load settlement curve is a minimum and for smaller size model footings, it is defined as the load corresponding to the point of break of the load settlement curve in a log-log plot. The available data are divided into two sets: training and validation. Eighty percent (i.e., 78) of the data are used for calibration and 20% (19) are used for validation. The representative set of patterns for the training phase has been selected in such a way that it contains all the patterns including the maximum and minimum values of all the input and output data. In the present study, the available data are randomly divided into training and validation sets in such a way that they are representative of same statistical population. Once training has been successfully accomplished, the performance of the model is tested.

n NN model development

The basic strategy for developing a neural-based model of material behavior is to train a neural network on the results of a series of experiments on material. If the experimental results contain the relevant information about the material behavior, then the trained neural network would contain sufficient information about the material behavior to qualify as a material model. Such a trained neural network not only would be able to reproduce the experimental results it was trained on, but through its generalization capability should be able to approximate the results of other experiments [25].

The feed-forward multilayer perceptron (MLP) is used in the present study; the description of which can be found in many publications [26,27]. A back propagation MLP with one hidden layer has proven to be capable of providing accurate approximation of any continuous function provided there are sufficient hidden nodes [28]. Hence, one hidden layer is used for the present study. As there are five input variables and one output variable, five nodes in the input layer and one node in the output layer are used. Further, NNs are very sensitive to the number of nodes in the hidden layer. Too many neurons in the hidden layer can lead to over fitting, i.e., the training data will be well modeled and the sum of the squared errors will be small, but the network will be modeling the noise in the data as well as the trends. Therefore, the network will not generalize well on the testing data. A common heuristic approach to avoid over fitting is early stopping. This approach involves monitoring the generalization error and stopping training when the minimum validation error is observed. However, some care is needed when to stop, since the validation error surface may have local minima or long flat regions preceding a steep drop-off [29].

Table 1 The data used for developing the model

Case No.	Source	B (m)	D (m)	L/B	γ_d or γ' (kN/m ³)	ϕ (deg)	q_u (kPa)
1	Muhs et al. [21]	0.6	0.3	2	9.85	34.9	270
2		0.6	0	2	10.2	37.7	200
3		0.6	0.3	2	10.2	37.7	570
4		0.6	0	2	10.85	44.8	860

5		0.6	0.3	2	10.85	44.8	1760
6	WeiB [20]	0.5	0	1	10.2	37.7	154
7		0.5	0	1	10.2	37.7	165
8		0.5	0	2	10.2	37.7	203
9		0.5	0	2	10.2	37.7	195
10		0.5	0	3	10.2	37.7	214
11		0.52	0	3.85	10.2	37.7	186
12		0.5	0.3	1	10.2	37.7	681
13		0.5	0.3	2	10.2	37.7	542
14		0.5	0.3	2	10.2	37.7	530
15		0.5	0.3	3	10.2	37.7	402
16		0.52	0.3	3.85	10.2	37.7	413
17	Muhs and WeiB [19]	0.5	0	1	11.7	37	111
18		0.5	0	1	11.7	37	132
19		0.5	0	2	11.7	37	143
20		0.5	0.013	1	11.7	37	137
21		0.5	0.029	4	11.7	37	109
22		0.5	0.127	4	11.7	37	187
23		0.5	0.3	1	11.7	37	406
24		0.5	0.3	1	11.7	37	446
25		0.5	0.3	4	11.7	37	322
26		0.5	0.5	2	11.7	37	565
27		0.5	0.5	4	11.7	37	425
28		0.5	0	1	12.41	44	782
29		0.5	0	4	12.41	44	797
30		0.5	0.3	1	12.41	44	1940
31		0.5	0.3	1	12.41	44	2266
32		0.5	0.5	2	12.41	44	2847
33		0.5	0.5	4	12.41	44	2033
34		0.5	0.49	4	12.27	42	1492
35		0.5	0	1	11.77	37	123
36		0.5	0	2	11.77	37	134
37		0.5	0.3	1	11.77	37	370
38		0.5	0.5	2	11.77	37	464
39		0.5	0	4	12	40	461
40		0.5	0.5	4	12	40	1140
41	Muhs and WeiB [22]	1	0.2	3	11.97	39	710
42		1	0	3	11.93	40	630
43	Briaud and Gibbens [23]	0.991	0.711	1	15.8	32	1773.7
44		3.004	0.762	1	15.8	32	1019.4
45		2.489	0.762	1	15.8	32	1158
46		1.492	0.762	1	15.8	32	1540
47		3.016	0.889	1	15.8	32	1161.2
48	Gandhi [24]	0.0585	0.029	5.95	15.7	34	58.5
49		0.0585	0.058	5.95	15.7	34	70.91
50		0.0585	0.029	5.95	16.1	37	82.5
51		0.0585	0.058	5.95	16.1	37	98.93
52		0.0585	0.029	5.95	16.5	39.5	121.5
53		0.0585	0.058	5.95	16.5	39.5	142.9
54		0.0585	0.029	5.95	16.8	41.5	157.5

Table 1 (continued)

Case No.	Source	B (m)	D (m)	L/B	γ_d or γ' (kN/m ³)	ϕ (deg)	q_u (kPa)
55		0.0585	0.058	5.95	16.8	41.5	184.9
56		0.0585	0.029	5.95	17.1	42.5	180.5
57		0.0585	0.058	5.95	17.1	42.5	211

58	0.094	0.047	6	15.7	34	74.7
59	0.094	0.094	6	15.7	34	91.5
60	0.094	0.047	6	16.1	37	104.8
61	0.094	0.094	6	16.1	37	127.5
62	0.094	0.047	6	16.5	39.5	155.8
63	0.094	0.094	6	16.5	39.5	185.6
64	0.094	0.047	6	16.8	41.5	206.8
65	0.094	0.094	6	16.8	41.5	244.6
66	0.094	0.047	6	17.1	42.5	235.6
67	0.094	0.094	6	17.1	42.5	279.6
68	0.152	0.075	5.95	15.7	34	98.2
69	0.152	0.15	5.95	15.7	34	122.3
70	0.152	0.075	5.95	16.1	37	143.3
71	0.152	0.15	5.95	16.1	37	176.4
72	0.152	0.075	5.95	16.5	39.5	211.2
73	0.152	0.15	5.95	16.5	39.5	254.5
74	0.152	0.075	5.95	16.8	41.5	285.3
75	0.152	0.15	5.95	16.8	41.5	342.5
76	0.152	0.075	5.95	17.1	42.5	335.3
77	0.152	0.15	5.95	17.1	42.5	400.6
78	0.094	0.047	1	15.7	34	67.7
79	0.094	0.094	1	15.7	34	90.5
80	0.094	0.047	1	16.1	37	98.8
81	0.094	0.094	1	16.1	37	131.5
82	0.094	0.047	1	16.5	39.5	147.8
83	0.094	0.094	1	16.5	39.5	191.6
84	0.094	0.047	1	16.8	41.5	196.8
85	0.094	0.094	1	16.8	41.5	253.6
86	0.094	0.047	1	17.1	42.5	228.8
87	0.094	0.094	1	17.1	42.5	295.6
88	0.152	0.075	1	15.7	34	91.2
89	0.152	0.15	1	15.7	34	124.4
90	0.152	0.075	1	16.1	37	135.2
91	0.152	0.15	1	16.1	37	182.4
92	0.152	0.075	1	16.5	39.5	201.2
93	0.152	0.15	1	16.5	39.5	264.5
94	0.152	0.075	1	16.8	41.5	276.3
95	0.152	0.15	1	16.8	41.5	361.5
96	0.152	0.075	1	17.1	42.5	325.3
97	0.152	0.15	1	17.1	42.5	423.6

The steps for developing NN models, as outlined by Maier and Dandy [30], are used as a guide in this work. These include the determination of model inputs and outputs, division and preprocessing of the available data, the determination of appropriate network architecture, optimization of the connection weights training, stopping criteria, and model validation. In this study, the general strategy adopted for finding the optimal parameters that control the training process is as follows.

For each trial number of hidden layer nodes, random initial weights and biases are generated. The neural network is then trained with different combinations of momentum terms and learning rates in an attempt to identify the NN model that performs best on the validation data. The momentum terms used in this study are 0.005, 0.01, 0.05, 0.1, 0.2, 0.4, 0.6, 0.8, 0.95 and 0.99, whereas the learning rates used are 0.05, 0.1, 0.15, 0.2, 0.4, 0.6, 0.8, 0.9 and 0.95. Since the back-propagation training algorithm uses a first-order gradient descent technique to adjust the connection weights, it may get trapped in a local minimum if the initial starting point in weight space is unfavorable.

Consequently, the model that has the optimum momentum term and learning rate is retrained a number of times with different initial weights and biases until no further improvement occurs.

n Model evaluation

Quantitative assessments of the degree to which the model simulations match the actual output are used to provide an evaluation of the model's predictive abilities. As a single evaluation measure is not available [31], a multi criteria assessment was performed in the current study with various goodness-of-fit statistics. These measures can be grouped into two types: relative and absolute. Relative goodness-of-fit measures are non-dimensional indices, which provide a relative comparison of the performance of one model against another. In contrast, absolute goodness-of-fit statistics are measured in the units of ultimate bearing capacity. The criteria that are employed for model evaluation are the coefficient of correlation (R), root-mean-square error (RMSE) between the actual and predicted values and the mean absolute error (MAE). The definition of these evaluation criteria is provided in Table 2.

Table 2 Performance evaluation criteria

Evaluation criteria	Definition
Coefficient of correlation (R)	$R = \frac{\dot{\mathbf{a}}_{i=1}^n (y_i^m - \overline{y^m})(y_i^c - \overline{y^c})}{\sqrt{\dot{\mathbf{a}}_{i=1}^n (y_i^m - \overline{y^m})^2} \sqrt{\dot{\mathbf{a}}_{i=1}^n (y_i^c - \overline{y^c})^2}}$
Root-mean-square error (RMSE)	$RMSE = \sqrt{\frac{\dot{\mathbf{a}}_{i=1}^n (y_i^m - y_i^c)^2}{n}}$
Mean absolute error (MAE)	$MAE = \frac{1}{n} \dot{\mathbf{a}}_{i=1}^n y_i^c - y_i^m $

Note: y_i^m and y_i^c are the measured and computed ultimate bearing capacity values, respectively, $\overline{y^m}$ and $\overline{y^c}$ are the mean of the measured and computed ultimate bearing capacity values corresponding to n patterns. Smith [32] suggested the following guide for the value of (r) between 0.0 and 1.0:

- $|r| \geq 0.8$ strong correlation exists between two sets of variables;
- $0.2 < |r| < 0.8$ correlation exists between the two sets of variables; and
- $|r| \leq 0.2$ weak correlation exists between the two sets of variables.

The RMSE is the most popular measure of error and has the advantage that large errors receive much greater attention than small errors. In contrast with RMSE, MAE eliminates the emphasis given to large errors. Both RMSE and MAE are desirable when the evaluated output data are smooth or continuous. The optimum model is a model with a highest value of (R) and a lowest value of (RMSE) and (MAE).

4. TRADITIONAL METHODS FOR ULTIMATE BEARING CAPACITY PREDICTION

Many theoretical methods for the prediction of ultimate bearing capacity of shallow foundations are presented in the literature. Among these, three are chosen for the purpose of assessing the relative performance of NN model. These include the methods proposed by Meyerhof [3], Vesic

[4] and Hansen [33]. These methods are used for comparison as they are commonly used for estimating the ultimate bearing capacity.

5. Results and Discussion

The impact of the number of hidden nodes, learning rate and momentum term on NN performance is shown in Figures 3,4, and 5 respectively. Fig. 3 shows that the network with three hidden layer nodes has the lowest prediction error. Figures 4 and 5 show that the best prediction was obtained with a momentum value of 0.95 and learning rate 0.8 respectively.

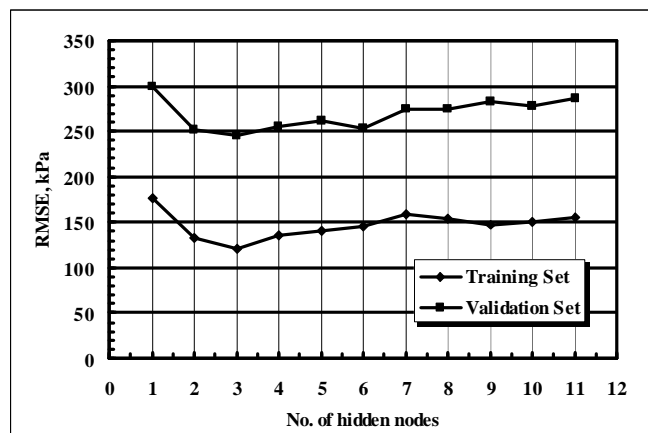


Fig. 3. Performance of neural network models with different hidden layer nodes (learning rate = 0.2 and momentum term = 0.8)

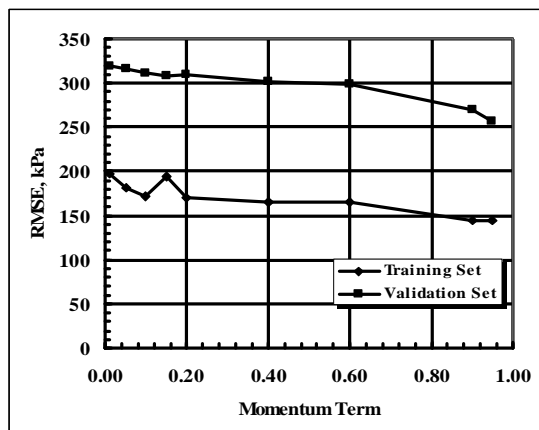


Fig.5 Effect of various momentum terms on neural network performance (hidden nodes = 3 and learning rate = 0.2)

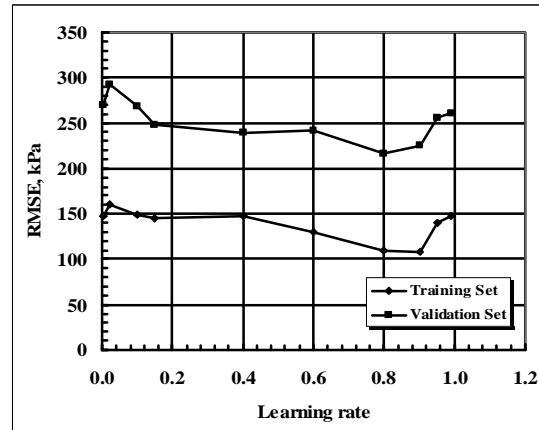


Fig. 6 Effect of various learning rates on neural network performance (hidden nodes = 3 and momentum term = 0.8)

The predictive performance of the optimal neural network model (i.e., three layer nodes, momentum value of 0.95, and learning rate of 0.8) is summarized in Table 3. The results indicate that the NN model performs well, with an R of 0.939, an RMSE of 158.67 kPa, and an MAE of 86.73 kPa for the validation set.

Table (3) Neural Network Results

Data set	R	RMSE (MPa)	MAE (MPa)
All data	0.983	101.23	59.74
Training	0.990	81.31	53.17
Validation	0.939	158.67	86.73

Comparisons of the results predicted using the NN and the measured values of bearing capacity are presented in Fig. 7, which shows that the NN model performs reasonably for all data, Training and Validation data used in this work.

The values of performance measures off NN model and bearing capacity theories for all data set are summarized in Table 4. The RMSE, and MAE values for the NN model are less than those for the traditional theories chosen in this work, while R are higher. Comparisons of the predicted values by traditional theories and the measured values of bearing capacity are presented in Fig. 8, which show that R^2 are higher for NN model than those for traditional theories. This indicates that the performance of NN model is better than the theoretical methods.

Table (4) Comparison of performance measures in model predicted bearing capacity values and traditional theories for all data set

Performance measures	NN Model	Terzaghi	Meyerhof	Hansen
R	0.983	0.902	0.939	0.942
RMSE (kPa)	101.23	340.38	188.26	295.743
MAE (kPa)	59.74	153.51	100.66	148.514

6-Conclusions

This paper deals with the problem of prediction of the ultimate bearing capacity of shallow foundations on cohesionless soil. The results indicate that the NN model is able to predict well the ultimate bearing capacity of shallow foundations. The model performs better than the theoretical methods. This was evidenced by the performance measures used for evaluating the models. Also, the advantage of these soft computing techniques is that they can be updated easily, as and when new data become available avoiding expertise and time needed to update the old design aid or equation and/or propose a new equation.

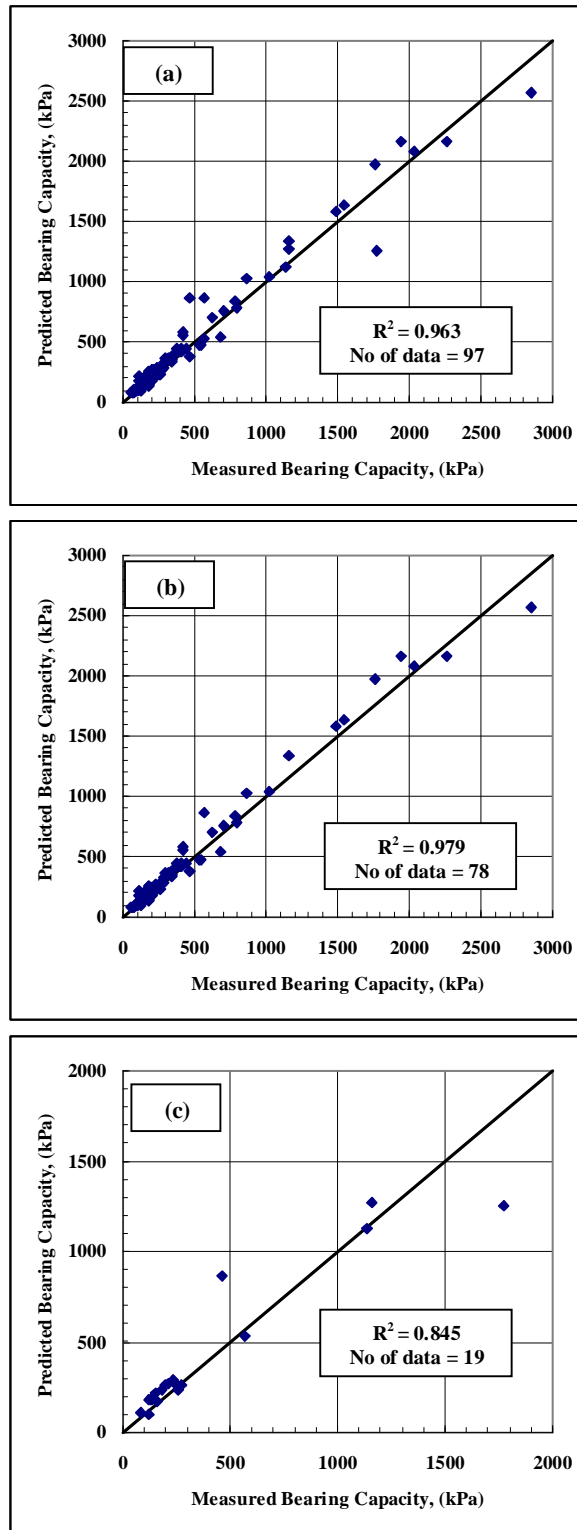


Fig. 7 Comparison of Predicted and Measured Bearing Capacity for
 (a) All data (b) Training data (c) Validation data

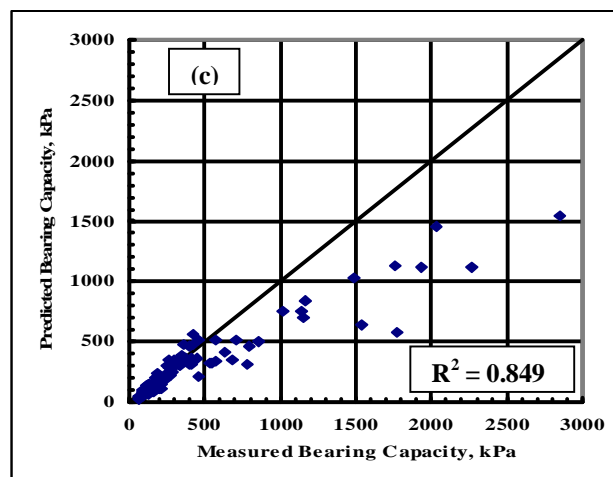
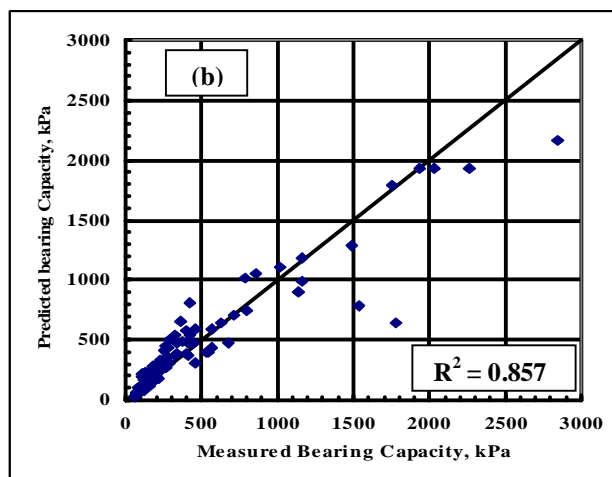
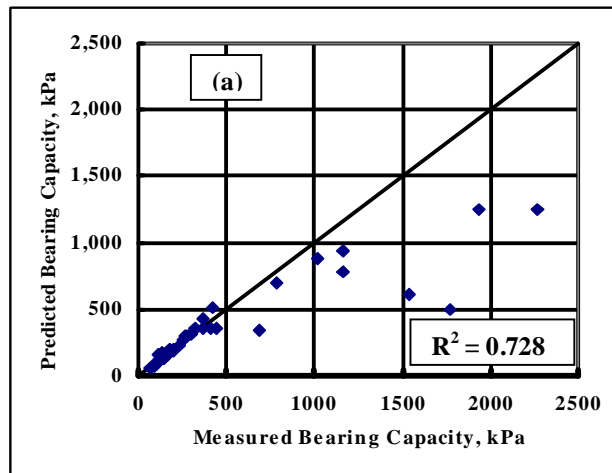


Fig. 8 Comparison of Measured and Predicted Bearing capacity by
 (a) Terzaghi (b) Meyerhof (c) Hansen theories

7. references

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