# Behavior of Multi-Layer Composite Beams with Partial Interaction Part II

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**الخلاصة**

تعتبر المنشآت ذات المقاطع متعددة الطبقات، والتي لها أبعاد وخواص مواد مختلفة، من المنشآت المهمة والتي تزايد الاهتمام بها وباستخداماتها مخصوصا" في الصناعات البحرية والطائرات والكثير من المنشآت المدنية . في هذا البحث تم اشتقاق معادلات تحقق متطلبات التوازن والتوافق للعتبات ذات مقاطع متعددة الطبقات ومختلفة من ناحية الخواص الهندسية والأبعاد ، هذه المعادلات تأخذ بنظر الاعتبار الإزاحات الأفقية والإزاحات العمودية لكل طبقة. إن التحليل يستند على الأساس الذي اعتمد من قبل روبرت والخاص بالعتبات ذات الطبقتين . تم تحليل عتبات متكونة من مقاطع ذات ثلاثة ، أربعة وخمسة طبقات مختلفة الأبعاد والخواص الهندسية. تم اشتقاق علاقات عامة لمعادلات التوازن والتوافق لأي عتبة بسيطة الإسناد ولأي مقطع بغض النظر عن عدد الطبقات المكون لها ، أبعادها أو رباطات القص .

**ABSTRACT**:

In this study an attempt is made to derive governing equations satisfying equilibrium and compatibility, for multi-layer composite beams with different layers, materials properties and dimensions for linear material and shear connector behavior in which the slip (horizontal displacement) and uplift force (vertical displacement) are taken into consideration. The analysis led to a set of number differential equations containing derivatives of the fourth and third order, number of these equations depending on number of layers forming the beam section. The theory developed for three, four, and five layers. A general formula were derived to find the governing equations (compatibility and equilibrium equations) for any layered composite beam.

## 1. INTRODUCTION:

Composite construction has been widely used for building construction. A perfect connection between the components of composite elements exists only theoretically. Experimental investigation has shown that significant slip occurs at the interface between these components, even when a large number of connectors are proved. The modification in the behavior of a composite beam by the presence of slip was illustrated by analysis conducted by many researchers. These analyses led to differential equations (number of these equations depending on the degree of freedom) that are to be solved fresh for each type of loading and the variation in dimensions or properties of beams. The first interaction theory that takes account of slip effects was initially formulated by Newmark [1], based on elastic analysis of composite beams assuming linear material and shear connector behavior. Adekola [2] present different model based on interaction theory, which takes account slip, uplift and friction effect. Using the same element presented by Newmark, Johnson [3] in 1975 proposed a partial interaction theory for simply supported beams, in which the analysis was based on elastic theory. The composite beam was assumed to be in linear elastic materials. Roberts [4] presented an approach for the analysis of composite beam with partial interaction, in which the basic equilibrium and compatibility equations were formulated in terms of four independent variables, i.e. the axial displacements of the concrete and steel and the deflections of the two layers. Linear elastic materials and shear connector behavior were assumed with the concrete remaining uncracked, and both the slip and separation at the interface were incorporated.

In engineering applications, layered systems of various materials are used to fabricate beams, plates and shells. The procedures commonly employed to analyze such systems are based on the assumption of rigid interconnection between layers. If the layers are fastened together with strong adhesives as in most of the laminated plastics as well as in welded assemblies, the assumption of rigid interconnection between layers is reasonable. In some widely used systems, however, such as in composite steel – concrete beams and especially in layered wood construction connected with nails, the later assumption is questionable. In the past, in the analysis of such problems, only limited consideration has been given to the effects of the interlayer movements, which occur as a result of deformation at the connectors. This interlayer movements or slip between layers can significantly affect overall behavior of a structure [5-12].

Laminated composite beams are very important types of construction in which the cross-section forms of different layers with different dimensions and material properties. The derivation deals with beams consist of three, four and five layers in different materials, different dimensions, different shear stiffness and normal stiffness for connectors. The analysis leads to a set of basic equilibrium and compatibility equations that were formulated in terms of displacements (horizontal and vertical) of each layer. These differential equations were expressed in finite difference form, and the resulting simultaneous algebraic equations were solved numerically.

## 2. Analytical solution for composite layered beam with partial interaction (three layers).

The basic concepts of composite beams of three layers, and linear behavior connected by the connectors, have been discussed in Ref.[13]

## 3. Analytical solution for composite layered beam with partial interaction (four layers).

In this section, the model, consists of four different layers, different materials and shear and normal stiffness. The analysis leads to a set of eight basic equilibrium and compatibility equations formulated in terms of displacements (horizontal and vertical) of each layer. These differential equations were expressed in finite difference form, and the resulting simultaneous algebraic equations were solved numerically.

### 3.1 Equilibrium

An element of a composite of four layers, length $dx$, shown in Figure (2). Subjected to moments, M, shear forces, V, and axial forces F. Subscripts a, b, c and d denote four layers from upper to lower layer, and the local x- axes pass through the centroids of the materials. If the beam is subjected to distributed load $r$ per unit length, vertical equilibrium of the whole element gives:

$$dV_a + dV_b + dV_c + dV_d = r dx \quad ..(1)$$

Dividing equation (1) by $dx$ and taking a limit as $dx$ tends to zero which gives:

$$V_{a,x} + V_{b,x} + V_{c,x} + V_{d,x} = r \quad ..(2)$$

This subscript x denotes differentiation. For live load only ($r$) equal to ($r_i$), for live load and dead load, $r$ is equal to:

$$r = r_i + r_a + r_b + r_c + r_d \quad ..(3)$$

In which, $r_a, r_b, r_c$ and $r_d$ are the distributed self weight for the four layers. Loads due to the removal of props used during construction should be considered as live loads.

Taking moments about the origin of coordinates in the upper layer which gives:

$$dM_a + dM_b + dM_c + dM_d = (V_a + V_b + V_c + V_d).dx + (dV_a + dV_b + dV_c + V_d).\frac{dx}{2} + dF_b.d_1 + dF_c.(d_1 + d_2) + dF_d.(d_1 + d_2 + d_3) \quad ..(4)$$

Where $d_1, d_2$ and $d_3$ are the distance between the centroids of any successive two cross sections.

After neglecting the second order terms and dividing by $dx$ equation (4) becomes:

$$M_{a,x} + M_{b,x} + M_{c,x} + M_{d,x} = V_a + V_b + V_c +_d + F_{b,x}.d_1 + F_{c,x}.(d_1 + d_2) + F_{d,x}(d_1 + d_2 + d_3) \quad ..(5)$$

Differentiating equation (5) gives:

$$M_{a,xx} + M_{b,xx} + M_{c,xx} + M_{d,xx} = V_{a,x} + V_{b,x} + V_{c,x} + V_{d,x} + F_{b,xx}.d_1 + F_{c,xx}.(d_1 + d_2) + F_{d,xx}(d_1 + d_2 + d_3) \quad ..(6)$$

Substituting equation (2) into equation (6) gives:

$$M_{a,xx} + M_{b,xx} + M_{c,xx} + M_{d,xx} - F_{b,xx}.d_1 - F_{c,xx}.(d_1 + d_2) - F_{d,xx}(d_1 + d_2 + d_3) = r \quad ..(7)$$

Taking moments about the origin of coordinates in the second layer gives:

$$dM_a + dM_b + dM_c + dM_d = (V_a + V_b + V_c + V_d).dx + (dV_a + dV_b + dV_c + V_d).\frac{dx}{2} - dF_a.d_1 + dF_c.d_2 + dF_d.(d_2 + d_3) \quad ..(8)$$

After neglecting the second order terms and dividing by $dx$ equation (8) becomes:

$$M_{a,x} + M_{b,x} + M_{c,x} + M_{d,x} = V_a + V_b + V_c + V_d - F_{a,x}.d_1 + F_{c,x}.d_2 + F_{d,x}(d_2 + d_3) \quad ..(9)$$

Differentiating equation (9) gives:

$$M_{a,xx} + M_{b,xx} + M_{c,xx} + M_{d,xx} = V_{a,x} + V_{b,x} + V_{c,x} + V_{d,x} - F_{a,xx}.d_1 + F_{c,xx}.d_2 + F_{d,xx}(d_2 + d_3) \quad ..(10)$$

Substituting equation (2) into (10) gives:

$$M_{a,xx} + M_{b,xx} + M_{c,xx} + M_{d,xx} + F_{a,xx}.d_1 - F_{c,xx}.(d_2) - F_{d,xx}(d_2 + d_3) = r \quad ..(11)$$

Taking moments about the origin of coordinates in the third layer gives:

$$dM_a + dM_b + dM_c + dM_d = (V_a + V_b + V_c + V_d).dx + (dV_a + dV_b + dV_c + V_d).\frac{dx}{2} - dF_a.(d_1 + d_2) - dF_b.d_2 + dF_d.d_3 \quad ..(12)$$

After neglecting the second order terms and dividing by $dx$ equation (12) becomes:

$$M_{a,x} + M_{b,x} + M_{c,x} + M_{d,x} = V_a + V_b + V_c + V_d - F_{a,x}.(d_1 + d_2) - F_{b,x}.d_2 + F_{d,x}d_3 \quad ..(13)$$

Differentiating equation (13) gives:

$$M_{a,xx} + M_{b,xx} + M_{c,xx} + M_{d,xx} = V_{a,x} + V_{b,x} + V_{c,x} + V_{d,x} - F_{a,xx}.(d_1 + d_2) - F_{b,xx}.d_2 + F_{d,xx}d_3 \quad ..(14)$$

Substituting equation (2) into (14) gives:

$$M_{a,xx} + M_{b,xx} + M_{c,xx} + M_{d,xx} + F_{a,xx}.(d_1 + d_2) + F_{b,xx}.d_2 - F_{d,xx} d_3 = r \quad ..(15)$$

For equilibrium of the composite element, shown in Figure (2), in the x-direction gives:

$$(dF_a + F_a) + (dF_b + F_b) + (dF_c + F_c) + (dF_d + F_d) - (F_a + F_b + F_c + F_d) = 0 \quad ..(16)$$

$$dF_a + dF_b + dF_c + dF_d = 0 \quad ..(17)$$

Dividing equation (17) by $dx$ gives:

$$F_{a,x} + F_{b,x} + F_{c,x} + F_{d,x} = 0 \quad ..(18)$$

***Equation (7), (11), (15) and (18) are the four basic equilibrium equations required for the complete solution.***

## 3.2 Compatibility

Assuming plane sections within each material remains plane, the total displacement of the upper layer in the x-direction at the interface, denoted by $U_{ati}$, is given by:

$$U_{ati} = u_a - z_{ai}.w_{a,x} \quad ..(19)$$

In which $z_{ai}$ is the z-coordinate of the interface relative to the local x-z axes and, $u_a$ and $w_a$ are the displacements of the upper layer in the x and z directions. Similarly for the other three layers:

$$U_{bti} = u_b - z_{bi}.w_{b,x} \quad ..(20)$$

$$U_{cti} = u_c - z_{ci}.w_{c,x} \quad ..(21)$$

$$U_{dti} = u_d - z_{di}.w_{d,x} \quad ..(22)$$

The slip, $U_{ab}$, at the interface between the first two layers is denoted as the relative displacement in the x-direction of initially adjacent particles, as shown in Figure (2). Hence:

$$U_{ab} = U_{ati} - U_{bti} \quad ..(23)$$

And between the other layers:

$$U_{bc} = U_{bti} - U_{cti} \quad ..(24)$$

$$U_{cd} = U_{cti} - U_{dti} \quad ..(25)$$

Combining Equations from (19) to (22) into equations from (23) to (25) gives:

$$U_{ab} = (u_a - z_{ai}.w_{a,x}) - (u_b - z_{bi}.w_{b,x}) \quad ..(26)$$

$$U_{bc} = (u_b - z_{bi}.w_{b,x}) - (u_c - z_{ci}.w_{c,x}) \quad ..(27)$$

$$U_{cd} = (u_c - z_{ci}.w_{c,x}) - (u_d - z_{di}.w_{d,x}) \quad ..(28)$$

If the shear stiffness of the joint per unit length between the upper two layers is denoted by $k_{s1}$, the shear force per unit length at the interface $q_1$ is given by:

$$q_1 = k_{s1}.U_{ab} \quad ..(29)$$

And between the other layers:

$q_2 = k_{s2}.U_{bc}$ ..(30)

$q_3 = k_{s3}.U_{cd}$ ..(31)

And considering the equilibrium of the upper layer in the x-direction gives:

$F_{a,x} = q_1 = k_{s1}.U_{ab}$ ..(32)

And considering the equilibrium of the second layer in the x-direction gives:

$F_{b,x} = q_2 - q_1$ ..(33)

$F_{b,x} = k_{s2}.U_{bc} - k_{s1}.U_{ab}$ ..(34)

$F_{a,x} + F_{b,x} = q_2 = k_{s2}.U_{bc}$ ..(35)

$F_{a,x} + F_{b,x} + F_{c,x} = q_3 = k_{s3}.U_{cd}$ ..(36)

Substituting for $U_{ab}$ from equation (26) into (32) gives:

$F_{a,x} - k_{s1}[(u_a - z_{ai}.w_{a,x}) - (u_b - z_{bi}.w_{b,x})] = 0$ ..(37)

Substituting for $U_{bc}$ from equation (26) and (27) into (35) gives:

$F_{a,x} + F_{b,x} - k_{s2}[(u_b - z_{bi}.w_{b,x}) - (u_c - z_{ci}.w_{c,x})] = 0$ ..(38)

Substituting for $U_{bc}$ from equation (28) into (36) gives:

$F_{a,x} + F_{b,x} + F_{c,x} - k_{s3}[(u_c - z_{ci}.w_{c,x}) - (u_d - z_{di}.w_{d,x})] = 0$ ..(39)

The separation at the interface between the first upper layers, $w_{ba}$ is the relative displacement in the z-direction of initially adjacent, as shown in Figure (2-c) is given by:

$W_{ba} = w_b - w_a$ ..(40)

The separation at the interface between other layers $w_{cb}$ and $w_{dc}$ is given by:

$W_{cb} = w_c - w_b$ ..(41)

$W_{dc} = w_d - w_c$ ..(42)

If $P_1$ denotes the normal force per unit length at the interface, equilibrium for the first layer in the z-direction is given by:

$V_{a,x} = r_i + r_a + P_1$ ..(43)

If $P_2$ denotes the normal force per unit length at the interface, equilibrium for the second layer in the z-direction is given by:

$V_{b,x} = P_2 - P_1 + r_b$ ..(44)

$V_{a,x} + V_{b,x} = P_2 + r_b + r_a + r_i$ . .(45)

If $P_3$ denotes the normal force per unit length at the interface, equilibrium For the third layer in the z-direction is given by:

$V_{c,x} = P_3 - P_2 + r_c$ ..(46)

Consider the moment equilibrium of the upper layer about the origin of coordinates which gives;

$V_a = M_{a,x} + q_1.z_{ai}$ . .(47)

Consider the moment equilibrium of the second layer about the origin of coordinates which gives:

$$V_b = M_{b,x} + q_2.z_{bi} - q_1.z_{bi} \quad ..(48)$$

Consider the moment equilibrium of the third layer about the origin of coordinates which gives:

$$V_c = M_{c,x} + q_3.z_{ci} - q_2.z_{ci} \quad ..(49)$$

Differentiating equation (47), (48) and (49) gives:

$$V_{a,x} = M_{a,xx} + q_{1,x}.z_{ai} \quad ..(50)$$

$$V_{b,x} = M_{b,xx} + q_{2,x}.z_{bi} - q_{1,x}.z_{bi} \quad ..(51)$$

$$V_{c,x} = M_{c,xx} + q_{3,x}.z_{ci} - q_{2,x}.z_{ci} \quad ..(52)$$

Differentiating equation (32), (35) and (36) with respect to (x) gives:

$$F_{a,xx} = q_{1,x} \quad ..(53)$$

$$F_{a,xx} + F_{b,xx} = q_{2,x} \quad ..(54)$$

$$F_{a,xx} + F_{b,xx} + F_{c,xx} = q_{3,x} \quad ..(55)$$

Substituting equations (50) to (52) into (53) to (55) gives:

$$V_{a,x} = M_{a,xx} + F_{a,xx}.z_{ai} \quad ..(56)$$

$$V_{b,x} = M_{b,xx} + F_{b,xx}.z_{bi} \quad ..(57)$$

$$V_{c,x} = M_{c,xx} + F_{c,xx}.z_{ci} \quad ..(58)$$

Substituting equations (44) and (46) into equation from (56) to (58) gives:

$$M_{b,xx} + F_{b,xx}.z_{bi} = P_2 - P_1 + r_b \quad ..(59)$$

$$M_{c,xx} + F_{c,xx}.z_{ci} = P_3 - P_2 + r_c \quad ..(60)$$

If the normal stiffness of the joint per unit length between the upper layers, is denoted by ($k_{n1}$) then:

$$P_1 = k_{n1}.W_{ba} = k_{n1}.(w_b - w_a) \quad ..(61)$$

If the normal stiffness of the joint per unit length between the middle layers, is denoted by ($k_{n2}$) then:

$$P_2 = k_{n2}.W_{cb} = k_{n2}.(w_c - w_b) \quad ..(62)$$

If the normal stiffness of the joint per unit length between the lower layers, is denoted by ($k_{n3}$) then:

$$P_3 = k_{n3}.W_{dc} = k_{n3}.(w_d - w_c) \quad ..(63)$$

Substituting equations (61), (62), and (63) into equations (59) and (60) gives:

$$M_{b,xx} + F_{b,xx}.z_{bi} + k_{n1}(w_b - w_a) - k_{n2}.(w_c - w_b) = r_b \quad ..(64)$$

$$M_{c,xx} + F_{c,xx}..z_{ci} + k_{n2}(w_c - w_b) - k_{n3}(w_d - w_c) = r_c \quad ..(65)$$

Subtracting equation (63) from (64) which gives:

$$M_{b,xx} + F_{b,xx}.z_{bi} + k_{n1}(w_b - w_a) - 2.k_{n2}.(w_c - w_b) - M_{c,xx} - F_{c,xx}.z_{ci} + k_{n3}(w_d - w_c) = r_b - r_c \quad ..(66)$$

***Equations, (37), (38), (39) and (66) are the four basic compatibility equations required for a complete solution.***

## 3.3 Basic differential equations

From the analytical model, the eight independent differential equations (equilibrium and compatibility), may be expressed in terms of displacement variables, ($u_a, w_a, u_b, w_b, u_c, w_c, u_d$) and ($w_d$) as follows:

Assuming plane sections within each material remains plane, the axial strain ($e$) can be expressed in terms of displacements ($u$ , $w$) relative to the local (x) and (z –axes), which are assumed to pass through the centroid of the four materials. Hence:

$$e_a = U_{at,x} = U_{a,x} - z_a.w_{a,xx} \quad ..(67)$$

$$e_b = U_{bt,x} = U_{b,x} - z_b.w_{b,xx} \quad ..(68)$$

$$e_c = U_{ct,x} = U_{c,x} - z_c.w_{c,xx} \quad ..(69)$$

$$e_d = U_{dt,x} = U_{d,x} - z_d.w_{d,xx} \quad ..(70)$$

Where subscripts (a, b, c) and (d) denote the layers. Subscript (x) denotes differentiation and (z) the distance form the origin of coordinates to the limits of the layers.

Stresses now can be related to strain via the material properties ($E_a, E_b, E_c$) and ($E_d$). For linear elastic materials ($E_a, E_b, E_c$) and ($E_d$) are constants, but for non-linear elastic and elasto-plastic materials, ($E_a, E_b, E_c$) and ($E_d$) are functions of strain.

The free strain due to shrinkage, temperature etc, is denoted by ($e_f$), while the strain induced during the construction sequence is denoted by ($e_r$). Hence, if ($u$) and ($w$) are assumed to exclude the displacements corresponding, to ($e_f$) and ($e_r$), the stresses in the layers are given by:

$$s_a = E_a(u_{a,x} - z_a.w_{a,xx} + e_{ra} - e_{fa}) \quad ..(71)$$

$$s_b = E_b(u_{b,x} - z_b.w_{b,xx} + e_b - e_{fb}) \quad ..(72)$$

$$s_c = E_c(u_{c,x} - z_c.w_{c,xx} + e_{rc} - e_{fc}) \quad ..(73)$$

$$s_d = E_d(u_{d,x} - z_d.w_{d,xx} + e_{rd} - e_{fd}) \quad ..(74)$$

The axial forces ($F_a, F_b, F_c$) and ($F_d$) and moments ($M_a, M_b, M_c$) and ($M_d$) are obtained by integrating the stresses, multiplying by the appropriate lever arms, ($z_a, z_b, z_c$) and ($z_d$), in the case of moments over the cross section area of each layer denoted by ($A_a, A_b, A_c$) and ($A_d$) Hence:

$$F_a = \int s_a.dA_a \quad ..(75)$$

$$F_b = \int s_b.dA_b \quad ..(76)$$

$$F_c = \int s_c.dA_c \quad ..(77)$$

$$F_d = \int s_d.dA_d \quad ..(78)$$

$$M_a = -\int s_a.z_a.dA_a \qquad ..(79)$$

$$M_b = -\int s_b.z_b.dA_b \qquad ..(80)$$

$$M_c = -\int s_c.z_c.dA_c \qquad ..(81)$$

$$M_d = -\int s_d.z_d.dA_d \qquad ..(82)$$

Substituting equations (71), (72), (73), (74) into equations from (75) to (82) which gives:

$$F_a = \int E_a.(u_{a,x} - z_a.w_{a,xx} + e_{ra} - e_{fa})dA_a \qquad ..(83)$$

$$F_b = \int E_b.(u_{b,x} - z_b.w_{b,xx} + e_{rb} - e_{fb})dA_b \qquad ..(84)$$

$$F_c = \int E_c.(u_{c,x} - z_c.w_{c,xx} + e_{rc} - e_{fc})dA_c \qquad ..(85)$$

$$F_d = \int E_d.(u_{d,x} - z_d.w_{d,xx} + e_{rd} - e_{fd})dA_d \qquad ..(86)$$

$$M_a = -\int E_a.(u_{a,x} - z_a.w_{a,xx} + e_{ra} - e_{fa}).z_a.dA_a \qquad ..(87)$$

$$M_b = -\int E_b.(u_{b,x} - z_b.w_{b,xx} + e_{rb} - e_{fb}).z_b.dA_b \qquad ..(88)$$

$$M_c = -\int E_c.(u_{c,x} - z_c.w_{c,xx} + e_{rc} - e_{fc}).z_c.dA_c \qquad ..(89)$$

$$M_d = -\int E_d.(u_{d,x} - z_d.w_{d,xx} + e_{rd} - e_{fd}).z_d.dA_d \qquad ..(90)$$

IF $E_a, E_b, E_c$, and $E_d$ are constants, integration of equations from (83) to (90) which gives:

$$F_a = E_a.A_a.u_{a,x} + E_a.(\bar{e}_{ra} - \bar{e}_{fa}) \qquad ..(91)$$

$$F_b = E_b.A_b.u_{b,x} + E_b.(\bar{e}_{rb} - \bar{e}_{fb}) \qquad ..(92)$$

$$F_c = E_c.A_c.u_{c,x} + E_c.(\bar{e}_{rc} - \bar{e}_{fc}) \qquad ..(93)$$

$$F_d = E_d.A_d.u_{d,x} + E_d.(\bar{e}_{rd} - \bar{e}_{fd}) \qquad ..(94)$$

$$M_a = E_a.I_a.w_{a,xx} \qquad ..(95)$$

$$M_b = E_b.I_b.w_{b,xx} \qquad ..(96)$$

$$M_c = E_c.I_c.w_{c,xx} \qquad ..(97)$$

$$M_d = E_d.I_d.w_{d,xx} \qquad ..(98)$$

In which, $I_a, I_b, I_c$, and $I_d$ are the second moments of area for the layers and $\bar{e}$ is the integration of the strain function over the cross sectional area of the corresponding materials.

***The following are the eight governing equations derived for four layers composite beam:***

$$M_{a,xx} + M_{b,xx} + M_{c,xx} + M_{d,xx} - F_{b,xx}.d_1 - F_{c,xx}.(d_1 + d_2) - F_{d,xx}(d_1 + d_2 + d_3) = r \qquad ..(99)$$

$$M_{a,xx} + M_{b,xx} + M_{c,xx} + M_{d,xx} + F_{a,xx}.d_1 - F_{c,xx}.(d_2) - F_{d,xx}(d_2 + d_3) = r \qquad .(100)$$

$$M_{a,xx} + M_{b,xx} + M_{c,xx} + M_{d,xx} + F_{a,xx}.(d_1 + d_2) + F_{b,xx}.d_2 - F_{d,xx} d_3 = r \quad ..(101)$$

$$F_{a,x} + F_{b,x} + F_{c,x} + F_{d,x} = 0 \quad ..(102)$$

$$F_{a,x} - k_{s1}[(u_a - z_{ai}.w_{a,x}) - (u_b - z_{bi}.w_{b,x})] = 0 \quad ..(103)$$

$$F_{a,x} + F_{b,x} - k_{s2}[(u_b - z_{bi}.w_{b,x}) - (u_c - z_{ci}.w_{c,x})] = 0 \quad ..(104)$$

$$F_{a,x} + F_{b,x} + F_{c,x} - k_{s3}[(u_c - z_{ci}.w_{c,x}) - (u_d - z_{di}.w_{d,x})] = 0 \quad ..(105\ )$$

$$M_{b,xx} + F_{b,xx}.z_{bi} + k_{n1}(w_b - w_a) - 2.k_{n2}.(w_c - w_b) - M_{c,xx} - F_{c,xx}.z_{ci} + k_{n3}(w_d - w_c) = r_b - r_c \quad ..(106)$$

After substituting equations from (91) to (98) into equations from (99) to (106) which gives:

$$E_a.I_a.w_{a,xxxx} + E_b.I_b.w_{b,xxxx} + E_c.I_c.w_{c,xxxx} + E_d.I_d.w_{d,xxxx} - E_b.A_b.d_1.u_{b,xxx} - E_b(\bar{e}_{rb} - \bar{e}_{fb})_{,xx}.d_1 - (d_1 + d_2).E_c.A_c.u_{c,xxx} - E_c.(d_1 + d_2)(\bar{e}_{rc} - \bar{e}_{fc})_{,xx} - (d_1 + d_2 + d_3).E_d.A_d.u_{d,xxx} - (d_d + d_2 + d_3).E_d.(\bar{e}_{rd} - \bar{e}_{fd})_{,xx} = r \quad ..(107)$$

$$E_a.I_a.w_{a,xxxx} + E_b.I_b.w_{b,xxxx} + E_c.I_c.w_{c,xxxx} + E_d.I_d.w_{d,xxxx} + E_a.A_a.d_1.u_{a,xxx} + E_a(\bar{e}_a - \bar{e}_a)_{,xx}.d_1 - d_2.E_c.A_c.u_{c,xxx} - E_c.d_{2.}(\bar{e}_{rc} - \bar{e}_{fc})_{,xx} - (d_2 + d_3).E_d.A_d.u_{d,xxx} - E_d.(d_2 + d_3)(\bar{e}_{rd} - \bar{e}_{fd})_{,xx} = r \quad ..(108)$$

$$E_a.I_a.w_{a,xxxx} + E_b.I_b.w_{b,xxxx} + E_c.I_c.w_{c,xxxx} + E_d.I_d.w_{d,xxxx} + E_a.A_a.(d_1 + d_2).u_{a,xxx} + E_a(\bar{e}_a - \bar{e}_a)_{,xx}.(d_1 + d_2) + d_2.E_b.A_b.u_{b,xxx} + E_b.d_{2.}(\bar{e}_b - \bar{e}_{bc})_{,xx} - d_3.E_d.A_d.u_{d,xxx} - E_d.d_3(\bar{e}_{rd} - \bar{e}_{fd})_{,xx} = r \quad .(109)$$

$$E_a.A_a.u_{a,xx} + E_a.(\bar{e}_{ra} - \bar{e}_{fa})_{,x} + E_b.A_b.u_{b,xx} + E_b(\bar{e}_{rb} - \bar{e}_{fb})_{,x} + E_c.A_c.u_{c,xx} + E_c.(\bar{e}_{rc} - \bar{e}_{fc})_{,x} + E_d.A_d.u_{d,xx} + E_d.(\bar{e}_d - \bar{e}_d)_{,x} = 0 \quad ..(110)$$

$$E_a.A_a.u_{a,xx} + E_a.(\bar{e}_{ra} - \bar{e}_{fa})_{,x} - k_{s1}[(u_a - z_{ai}.w_{a,x}) - (u_b - z_{bi}.w_{b,x})] = 0 \quad ..(111)$$

$$E_a.A_a.u_{a,xx} + E_a.(\bar{e}_{ra} - \bar{e}_{fa})_{,x} + E_b.A_b.u_{b,xx} + E_b(\bar{e}_{rb} - \bar{e}_{fb})_{,x} - k_{s2}[(u_b - z_{bi}.w_{b,x}) - (u_c - z_{ci}.w_{c,x})] = 0 \quad ..(112)$$

$$E_a.A_a.u_{a,xx} + E_a.(\bar{e}_{ra} - \bar{e}_{fa})_{,x} + E_b.A_b.u_{b,xx} + E_b(\bar{e}_{rb} - \bar{e}_{fb})_{,x} + E_c.A_c.u_{c,xx} + E_c(\bar{e}_{rc} - \bar{e}_{fc}) - k_{s3}[(u_c - z_{ci}.w_{c,x}) - (u_d - z_{di}.w_{d,x})] = 0 \quad ..(113)$$

$$E_b.I_b.w_{b,xxxx} + E_b.A_b.u_{b,xxx}.z_{bi} + E_b.z_{bi}(\bar{e}_{rb} - \bar{e}_{fb})_{,xx} - E_c.I_c.w_{c,xxxx} - E_c.A_c.u_{c,xxx}.z_{ci} - E_c.z_{ci}(\bar{e}_{rc} - \bar{e}_{fc}) + k_{n1}(w_b - w_a) - 2k_{n2}.(w_c - w_b) + k_{n3}(w_d - w_c) = r_b - r_c \quad ..(114)$$

## 3.4 Numerical solutions

Equations (107) to (114) contain derivative of third order in u and fourth order in w, which can be expressed in finite (central) difference form using five node points.

After expressing equations (107) to (114) in finite difference form, the complete solution system of algebraic equations, eight degrees of freedom per node, can be solved for the unknown displacements at the nodes, and it required two external nodes are required at each end of the beam.

## 3.5 Boundary conditions.

Solution of the resulting set of algebraic equations requires the specification of boundary conditions. In general, the equations contain a derivative of fourth order and required two external nodes to specify the boundary conditions at each end. However, if each external node is assigned eight degree of freedom per node, so sixteen boundary conditions are required for each end of the beam must be specified.

$w_d = 0$ at $x = 0$ when $x = L$ ..(115)

$w_{a,xx} = 0$ at $x = 0$ when $x = L$ .(116)

$w_{b,xx} = 0$ at $x = 0$ when $x = L$ ..(117)

$w_{c,xx} = 0$ at $x = 0$ when $x = L$ ..(118)

$w_{d,xx} = 0$ at $x = 0$ when $x = L$ ..(119)

$u_d = 0$ at $x = 0$ ..(120)

$u_{d,x} = 0$ at $x = L$ ..(121)

$u_{a,x} = 0$ at $x = 0$ when $x = L$ ..(122)

$u_{b,x} = 0$ at $x = 0$ when $x = L$ ..(123)

$u_{c,x} = 0$ at $x = 0$ when $x = L$ ..(124)

$V_a + V_b + V_c + V_d = R_r$ at $x = 0$ ..(125)

$V_a + V_b + V_c + V_d = R_l$ at $x = L$ ..(126)

$u_{a,xxxx} = 0$ at $x = 0$ when $x = L$ ..(127)

$u_{b,xxxx} = 0$ at $x = 0$ when $x = L$ ..(128)

$u_{c,xxxx} = 0$ at $x = 0$ when $x = L$ ..(129)

$u_{d,xxxx} = 0$ at $x = 0$ when $x = L$ ..(130)

$U_{ab,x} = 0$ at $x = 0$ when $x = L$ ..(131)

$U_{bc,x} = 0$ at $x = 0$ when $x = L$ ..(132)

Where ($R_r$) and ($R_l$) are the reactions at the supports, equation (125) and (126) express the conditions that the sum of the shear forces in the layers are equal to the support reaction ($R_r$) and ($R_l$). The forces ($V_a, V_b, V_c$) and ($V_d$) can be expressed in terms of displacement derivatives as follows, consider moment equilibrium of the upper layer about the origin of coordinate, Figure (2), which gives:

$$V_a = M_{a,x} + F_{a,x}.z_{ai} \quad ..(133)$$

Similarly, for second layer:

$$V_b = M_{b,x} + F_{b,x}.z_{bi} \quad ..(134)$$

And for other layers:

$$V_c = M_{c,x} + F_{c,x}.z_{ci} \quad ..(135)$$

$$V_d = M_{d,x} + F_{d,x}.z_{di} \quad ..(136)$$

Substituting the forces and moments in terms of derivatives from equation (91) to (98) into equations from (133) to (136), which gives:

$$V_a = E_a.I_a.w_{a,xxx} + E_a.A_a.z_{ai} + E_a.z_{ai}.(\bar{e}_{ra} - \bar{e}_{fa})_{,x} \quad ..(137)$$

$$V_b = E_b.I_b.w_{b,xxx} + E_b.A_b.z_{bi} + E_b.z_{bi}.(\bar{e}_{rb} - \bar{e}_{fb})_{,x} \quad ..(138)$$

$$V_c = E_c.I_c.w_{c,xxx} + E_c.A_c.z_{ci} + E_c.z_{ci}.(\bar{e}_{rc} - \bar{e}_{fc})_{,x} \quad ..(139)$$

$$V_d = E_d.I_d.w_{d,xxx} + E_d.A_d.z_{di} + E_d.z_{di}.(\bar{e}_{rd} - \bar{e}_{fd})_{,x} \quad ..(140)$$

And for the latest boundary conditions, substituting equation (27) into (132) which gives:

$$U_{bc,x} = (u_{b,x} - z_{bi}.w_{b,xx}) - (u_{c,x} - z_{ci}.w_{c,xx}) \quad ..(141)$$

But equation (4.259) into a finite difference forms, which gives:

$$\frac{1}{\Delta x^2}(u_{b_{n+1}} - 2.u_{b_n} + u_{b_{n-1}}) - \frac{z}{2.\Delta x^3}(.w_{b_{n+2}} - 2.w_{b_{n-1}} + 2.w_{b_{n-1}} - w_{b_{n-2}}) - \frac{1}{\Delta x^2}(u_{c_{n+1}} - 2.u_{c_n} + u_{c_{n-1}}) + \frac{1}{2.\Delta x^3}(.w_{c_{n+2}} - 2.w_{c_{n-1}} + 2.w_{c_{n-1}} - w_{c_{n-2}}) = 0 \quad (142)$$

The main equations after substituting the finite difference form become:

$$\frac{E_a.I_a}{\Delta x^4}(w_{a_{n+2w}} - 4.w_{a_{n+1}} + 6.w_{a_n} - 4.w_{a_{n-1}} + w_{a_{n-2}}) + \frac{E_{b.}.I_b}{\Delta x^4}(w_{b_{n+2}} - 4.w_{b_{+n1}} + 6.w_{b_n} - 4.w_{b_{n-1}} + w_{b_{n-2}}) + \frac{E_c.I_c}{\Delta x^4}(w_{c_{n+2}} - 4.w_{c_{n+1}} + 6.w_{c_n} - 4.w_{c_{n-1}} + w_{c_{n-2}}) + \frac{E_d.I_d}{\Delta x^4}(w_{d_{n+2}} - 4.w_{d_{n+1}} + 6w_{d_n} - 4w_{d_{n-1}} + w_{d_{n-2}}) - \frac{E_b.A_b.d_1}{2.\Delta x^3}(u_{b_{n+2}} - 2.u_{b_{n+1}} + 2.u_{b_{n-1}} - u_{b_{n-2}}) - E_b.d_1.(\bar{e}_{rb} - \bar{e}_{fb})_{,xx} - \frac{E_c.A_c(d_1+d_2)}{2.\Delta x^3}(u_{c_{n+2}} - 2.u_{c_{n+1}} + 2.u_{c_{n-1}} - u_{c_{n-2}}) - E_c.(d_1+d_2).(\bar{e}_{rc} - \bar{e}_{fc})_{,xx} - \frac{E_d.A_d(d_1+d_2+d_3)}{2\Delta x^3}(u_{d_{n+2}} - 2u_{d_{n+1}} + 2u_{d_{n-1}} - u_{d_{n-2}}) - E_d(d_1+d_2+d_3)(\bar{e}_{rd} - \bar{e}_{fd})_{,xx} = r \quad ....(143)$$

$$\frac{E_a.I_a}{\Delta x^4}(w_{a_{n+2w}} - 4.w_{a_{n+1}} + 6.w_{a_n} - 4.w_{a_{n-1}} + w_{a_{n-2}}) + \frac{E_{b.}.I_b}{\Delta x^4}(w_{b_{n+2}} - 4.w_{b_{+n1}} + 6.w_{b_n} - 4.w_{b_{n-1}} + w_{b_{n-2}}) + \frac{E_c.I_c}{\Delta x^4}(w_{c_{n+2}} - 4.w_{c_{n+1}} + 6.w_{c_n} - 4.w_{c_{n-1}} + w_{c_{n-2}}) + \frac{E_d.I_d}{\Delta x^4}(w_{d_{n+2}} - 4w_{d_{n+1}} + 6w_{d_n} - 4w_{d_{n-1}} + w_{d_{n-2}}) + \frac{E_a.A_a.d_1}{2.\Delta x^3}(u_{a_{n+2}} - 2.u_{a_{n+1}} + 2.u_{a_{n-1}} - u_{a_{n-2}}) + E_a.d_1.(\bar{e}_{ra} - \bar{e}_{fa})_{,xx} - \frac{E_c.A_c d_2}{2.\Delta x^3}(u_{c_{n+2}} - 2.u_{c_{n+1}} + 2.u_{c_{n-1}} - u_{c_{n-2}}) - E_c.d_2.(\bar{e}_{rc} - \bar{e}_{fc}) - \frac{E_d.A_d(d_2+d_3)}{2\Delta x^3}(u_{d_{n+2}} - 2u_{d_{n+1}} + 2u_{d_{n-1}} - u_{d_{n-2}}) - E._d(d_2+d_3)(\bar{e}_{rd} - \bar{e}_{fd})_{,xx} = r \quad ..\ \mathbf{(144)}$$

$$\frac{E_a.I_a}{\Delta x^4}(w_{a_{n+2w}}-4.w_{a_{n+1}}+6.w_{a_n}-4.w_{a_{n-1}}+w_{a_{n-2}})+\frac{E_{b.}.I_b}{\Delta x^4}(w_{b_{n+2}}-4.w_{b_{+n1}}+6.w_{b_n}$$
$$-4.w_{b_{n-1}}+w_{b_{n-2}})+\frac{E_c.I_c}{\Delta x^4}(w_{c_{n+2}}-4.w_{c_{n+1}}+6.w_{c_n}-4.w_{c_{n-1}}+w_{c_{n-2}})+\frac{E_d.I_d}{\Delta x^4}$$
$$(w_{d_{n+2}}-4w_{d_{n+1}}+6w_{d_n}-4w_{d_{n-1}}+w_{d_{n-2}})+\frac{E_a.A_a.(d_1+d_2)}{2.\Delta x^3}(u_{a_{n+2}}-2.u_{a_{n+1}}+2.u_{a_{n-1}}-u_{a_{n-2}}) \quad ..(145)$$
$$+E_a.(d_1+d_2).(\bar{e}_{ra}-\bar{e}_{fa})_{,xx}+\frac{E_b.A_b d_2}{2.\Delta x^3}(u_{b_{n+2}}-2.u_{b_{n+1}}+2.u_{b_{n-1}}-u_{b_{n-2}})+E_b.d_2.(\bar{e}_{rb}-\bar{e}_b)$$
$$-\frac{E_d.A_d d_3}{2\Delta x^3}(u_{d_{n+2}}-2u_{d_{n+1}}+2u_{d_{n-1}}-u_{d_{n-2}})-E._d\, d_3(\bar{e}_{rd}-\bar{e}_{fd})_{,xx}=r$$

$$\frac{E_a.A_a.}{\Delta x^2}(.u_{a_{n+1}}-2.u_{a_n}+.u_{a_{n-1}})+E_a.(\bar{e}_{ra}-\bar{e}_{fa})_{,xx}+\frac{E_b.A_b}{\Delta x^2}(u_{b_{n+1}}-2.u_{b_n}+u_{b_{n-1}})$$
$$+E_b.(\bar{e}_{rb}-\bar{e}_{fb})_{,x}+\frac{E_c.A_c}{.\Delta x^2}(u_{c_{n+1}}-2.u_{c_n}+u_{c_{n-1}})+E_c.(\bar{e}_{rc}-\bar{e}_{fc})_{,x}+ \quad ..(146)$$
$$\frac{E_d.A_d}{\Delta x^2}(u_{d_{n+1}}-2u_{d_n}+u_{d_{n-1}})+E_d(\bar{e}_{rd}-\bar{e}_{fd})_{,x}=0$$

$$\frac{E_a.A_a}{\Delta x^2}(.u_{a_{n+1}}-2.u_{a_n}+.u_{a_{n-1}})+E_a.(\bar{e}_{ra}-\bar{e}_{fa})_{,x}-k_{s1}.u_{a_n}+\frac{k_{s1}.z_{ai}}{2.\Delta x}(w_{a_{n+1}}-w_{a_{n-1}})+ \quad ..(147)$$
$$+k_{s1}.u_{b_n}-\frac{k_{s1}.z_{b1}}{2..\Delta x}(w_{b_{n+1}}-w_{b_{n-1}})=0$$

$$\frac{E_a.A_a.}{\Delta x^2}(.u_{a_{n+1}}-2.u_{a_n}+.u_{a_{n-1}})+E_a.(\bar{e}_{ra}-\bar{e}_{fa})_{,x}+\frac{E_b.A_b}{\Delta x^2}(u_{b_{n+1}}-2.u_{b_n}+u_{b_{n-1}}) \quad ..(148)$$
$$+E_b.(\bar{e}_{rb}-\bar{e}_{fb})_{,x}-k_{s2}.u_{b_n}+\frac{k_{s2}.z_{bi}}{2.\Delta x}(w_{b_{n+1}}-w_{b_{n-1}})+k_{s2}.u_{c_n}-\frac{k_{s2}.z_{ci}}{2.\Delta x}.(w_{c_{n+1}}-w_{c_{n-1}})=0$$

$$\frac{E_a.A_a.}{\Delta x^2}(.u_{a_{n+1}}-2.u_{a_n}+.u_{a_{n-1}})+E_a.(\bar{e}_{ra}-\bar{e}_{fa})_{,xx}+\frac{E_b.A_b}{\Delta x^2}(u_{b_{n+1}}-2.u_{b_n}+u_{b_{n-1}})$$
$$+E_b.(\bar{e}_{rb}-\bar{e}_{fb})_{,x}+\frac{E_c.A_c}{\Delta x^2}(u_{c_{n+1}}-2u_{c_n}+u_{c_{n-1}})+E_c(\bar{e}_{rc}-\bar{e}_{fc})_{,x} \quad ..(149)$$
$$-k_{s3}.u_{c_n}+\frac{k_{s3}.z_{ci}}{2.\Delta x}(w_{c_{n+1}}-w_{c_{n-1}})+k_{s3}.u_{d_n}-\frac{k_{s3}.z_{di}}{2.\Delta x}.(w_{d_{n+1}}-w_{d_{n-1}})=0$$

$$\frac{E_{b.}.I_b}{\Delta x^4}.(w_{b_{n+2}}-4.w_{b_{+n1}}+6.w_{b_n}-4.w_{b_{n-1}}+w_{b_{n-2}})+\frac{E_b.A_b.z_{bi}}{2.\Delta x^3}(u_{b_{n+2}}-2.u_{b_{n+1}}$$
$$+2.u_{b_{n-1}}-u_{b_{n-2}})+E_b.z_{bi}(\bar{e}_{rb}-\bar{e}_{fb})_{,xx}-\frac{E_c I_c}{\Delta x^4}(w_{c_{n+2}}-4w_{c_{n+1}}+6w_{c_n} \quad ..(150)$$
$$-4w_{c_{n-1}}+w_{c_{n-2}})-\frac{E_c.A_c.z_{ci}}{2\Delta x^3}(u_{c_{n+2}}-2u_{c_{n+1}}+2u_{c_{n-1}}-u_{c_{n-2}})-E_c.z_{ci}(\bar{e}_{rc}-\bar{e}_{fc})$$
$$+k_{n1}.(w_{b_n}-w_{a_n})-2k_{n2}.(w_{c_n}-w_{b_n})+k_{n3}(w_{d_n}-w_{c_n})=r_b-r_c$$

## 4.General formula

According to the governing equations obtained from the analytical model of three, four, and five layers, a general formula can be obtained by the following sequence:

$1.For.k = 1.to.n - 1$

$$\sum_{i=1}^{n} M_{i,xx} - \sum_{i=k+1}^{n} F_{i,xx}.(\sum_{j=k}^{i-1} d_j) + \sum_{i=k-1}^{0} F_{i,xx}.(\sum_{j=1}^{i} d_j) = r_l + \sum_{i=1}^{n} r_i \quad ..(151)$$

2.

$$\sum_{i=1}^{n} F_{i,x} = 0 \quad ..(152)$$

$3.For.k = 1.to.n - 1$

$$\sum_{i=1}^{k} F_{i,x} - [ks_k.U_{(k,k+1)}] = 0 \quad ..(153)$$

$$4.\sum_{k=2}^{n-1}\{(-1)^k [M_{k,xx} + F_{k,xx}.z_{ki}]\} + kn_1(w_2 - w_1) + \sum_{j=2}^{n-2} 2(-1)^{j+1}.k_{nj}.(w_{j+1} - w_j)]$$
$$- kn_{n-1}(w_n - w_{n-1}) = \sum_{k=2}^{n-1} (-1)^k r_k \quad ..(154)$$

## 5 Procedure of application

The following procedure is used for applications of the general formula introduced in section (4.5). Number of equations depends on the number of layers and equal to (n*2) where n is number of layers. For example, the governing equations for four layer composite beams equal to (8) equations and can be derived directly as follows;

Number of layers (n)=4

Number of equations=8

Variables k, I, j are counters

For k=1 to n-1

Equation (1)

k=1

n=4

$$M_{1,xx} + M_{2,xx} + M_{3,xx} + M_{4,xx} - F_{2,xx}.d_1 - F_{3,xx}.(d_1 + d_2)$$
$$- F_{4,xx}(d_1 + d_2 + d_3) = r \quad ..(155)$$

Equation (2)

k=2

n=4

$$M_{1,xx} + M_{2,xx} + M_{3,xx} + M_{4,xx} + F_{1,xx}.d_1 - F_{3,xx}.(d_2)$$
$$- F_{4,xx}(d_2 + d_3) = r \quad ..(156)$$

Equation (3)

k=3

n=4

$$M_{1,xx} + M_{2,xx} + M_{3,xx} + M_{4,xx} + F_{1,xx}.(d_1 + d_2) + F_{2,xx}.d_2 - F_{4,xx} d_3 = r \quad ..(157)$$

End loop

Equation (4)

$$F_{1,x} + F_{2,x} + F_{3,x} + F_{4,x} = 0 \quad ..(158)$$

For k=1 to n-1

Equation (5)

k=1

n=4

$$F_{1,x} - k_{s1}[(u_1 - z_{1i}.w_{1,x}) - (u_2 - z_{2i}.w_{2,x})] = 0 \quad ..(159)$$

Equation (6)

k=2

n=4

$$F_{1,x} + F_{2,x} - k_{s2}[(u_2 - z_{2i}.w_{2,x}) - (u_3 - z_{3i}.w_{3,x})] = 0 \quad ..(160)$$

Equation (7)

k=3

n=4

$$F_{1,x} + F_{2,x} + F_{3,x} - k_{s3}[(u_3 - z_{3i}.w_{3,x}) - (u_4 - z_{4i}.w_{4,x})] = 0 \quad ..(161)$$

End loop

Equation (8)

$$M_{2,xx} + F_{2,xx}.z_{2i} + k_{n1}(w_2 - w_1) - 2.k_{n2}.(w_3 - w_2) - M_{3,xx} - F_{3,xx}.z_{3i} + k_{n3}(w_4 - w_3) = r_2 - r_3 \quad ..(162)$$

Subscripts (1, 2, 3) and (4) represents layers (a, b, c) and (d) respectively.

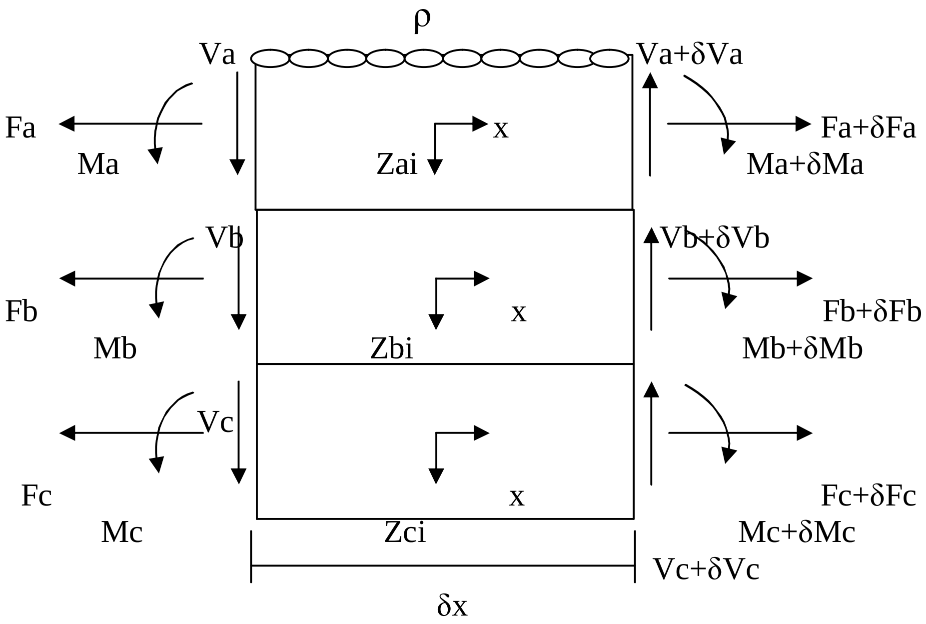


Figure (1-a) Composite layered beam

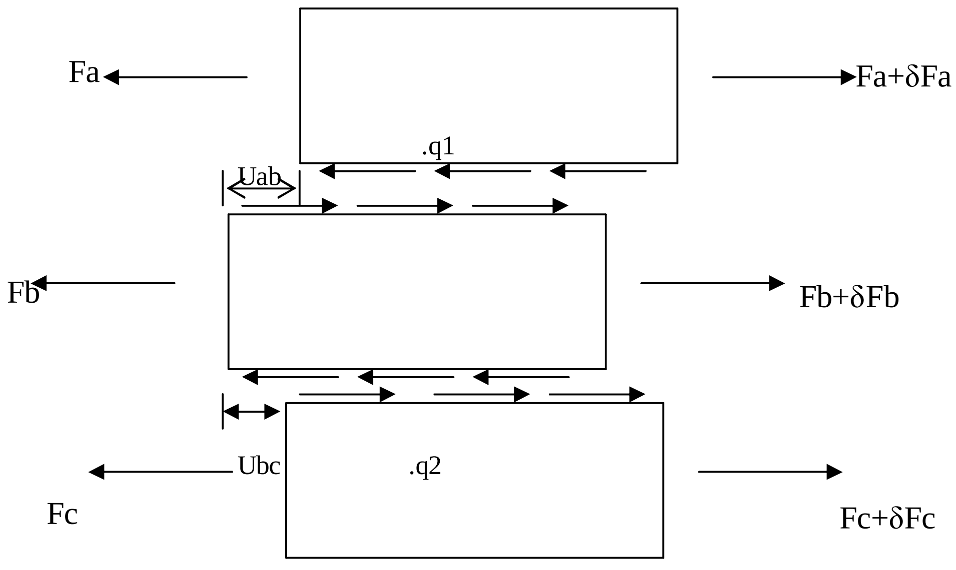


Figure (1-b) Composite layers element in Slip

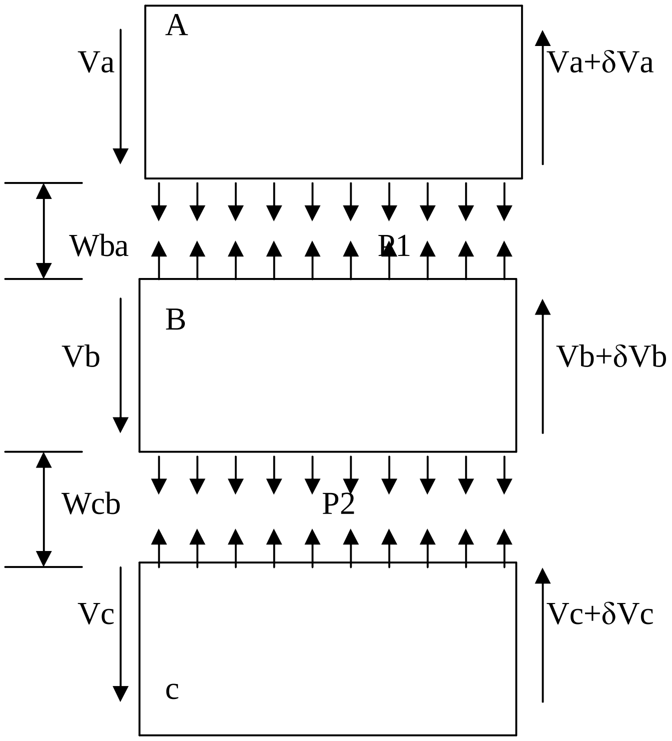


Figure (1-c) Composite layers in separation

Figure (1) Composite three layers element

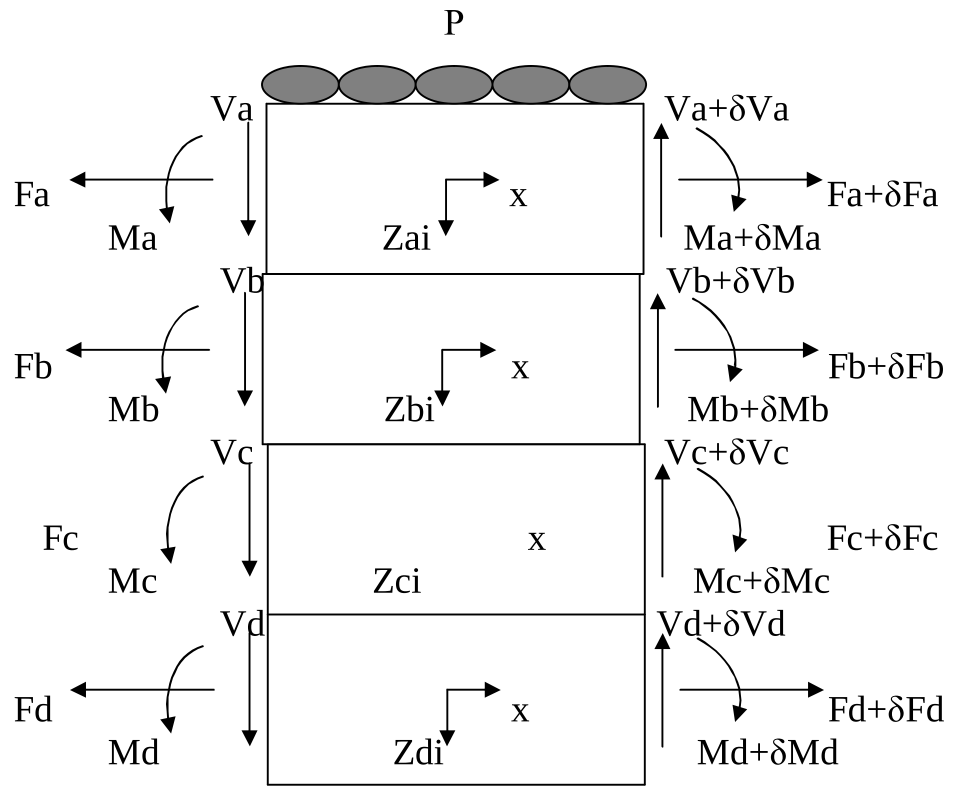


Figure (2-a) Composite four layers element

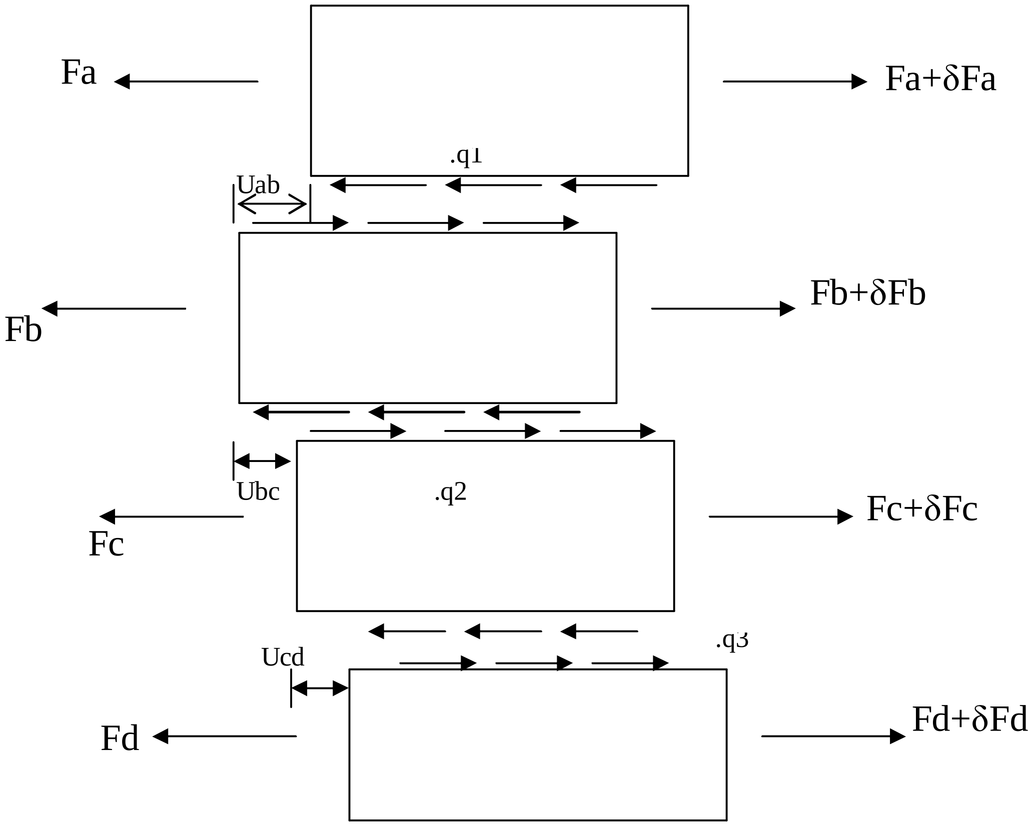


Figure (2-b) Composite four layers element in slip

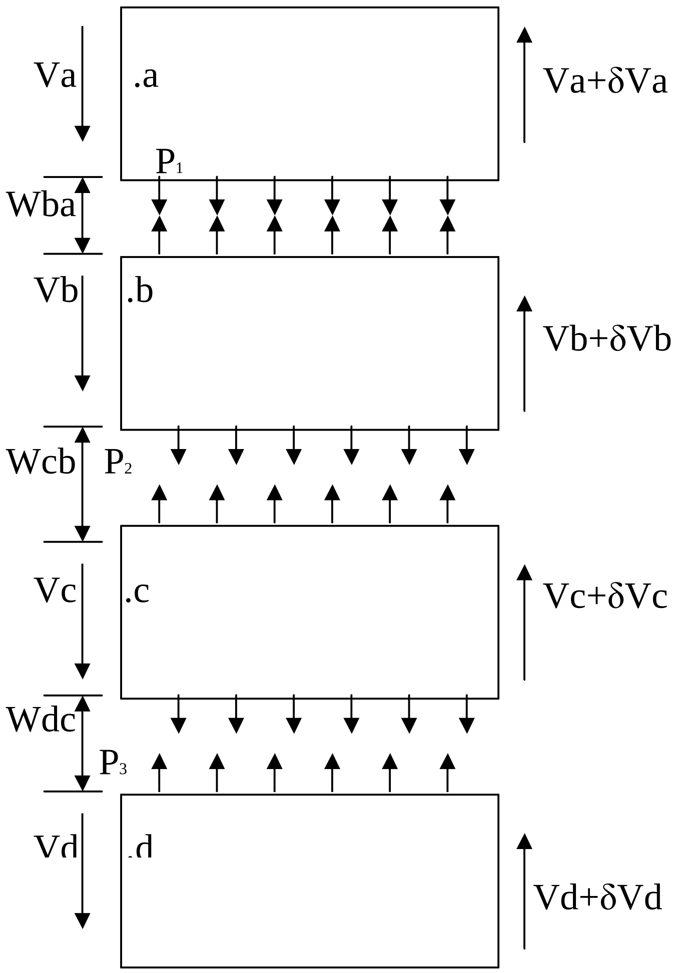


Figure (2-c) Composite four layers in separation

Figure (2) Composite four layers element.

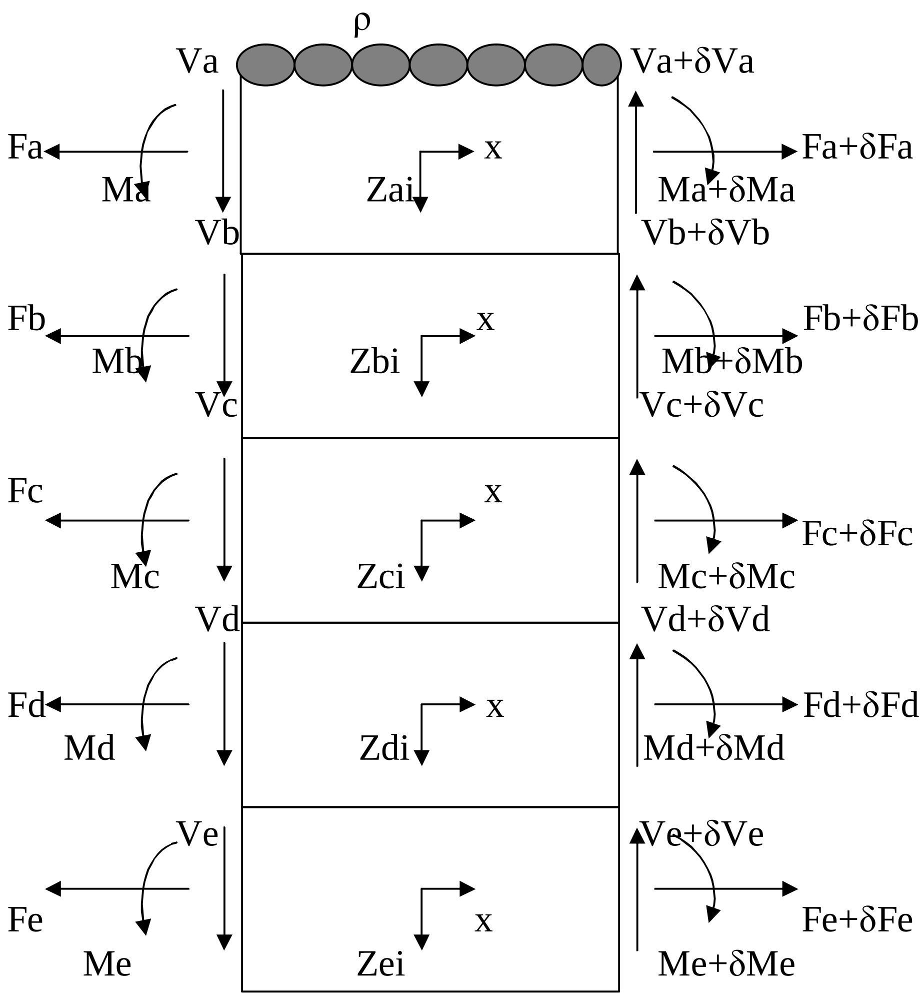


Figure (3-a) Composite five layers element

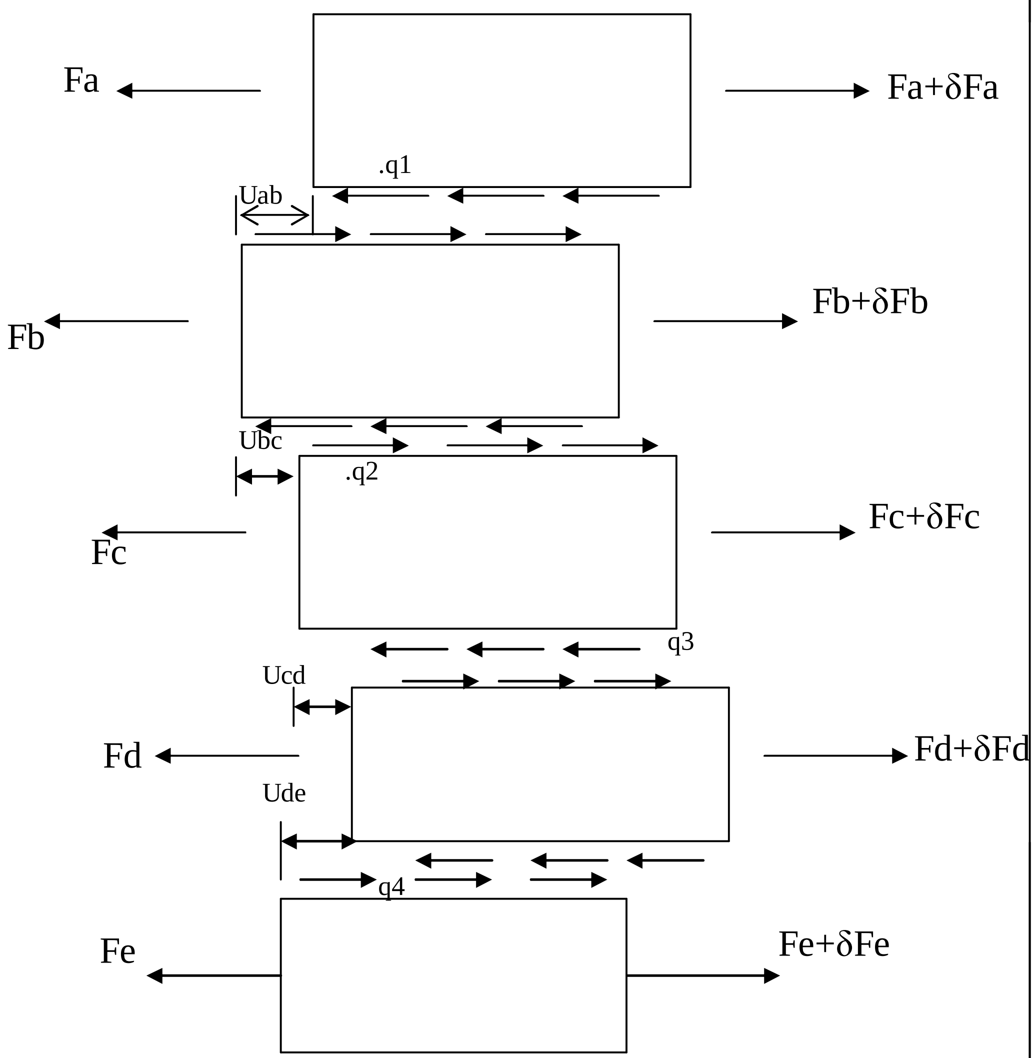


Figure (3-b) Composite five layers element in Slip

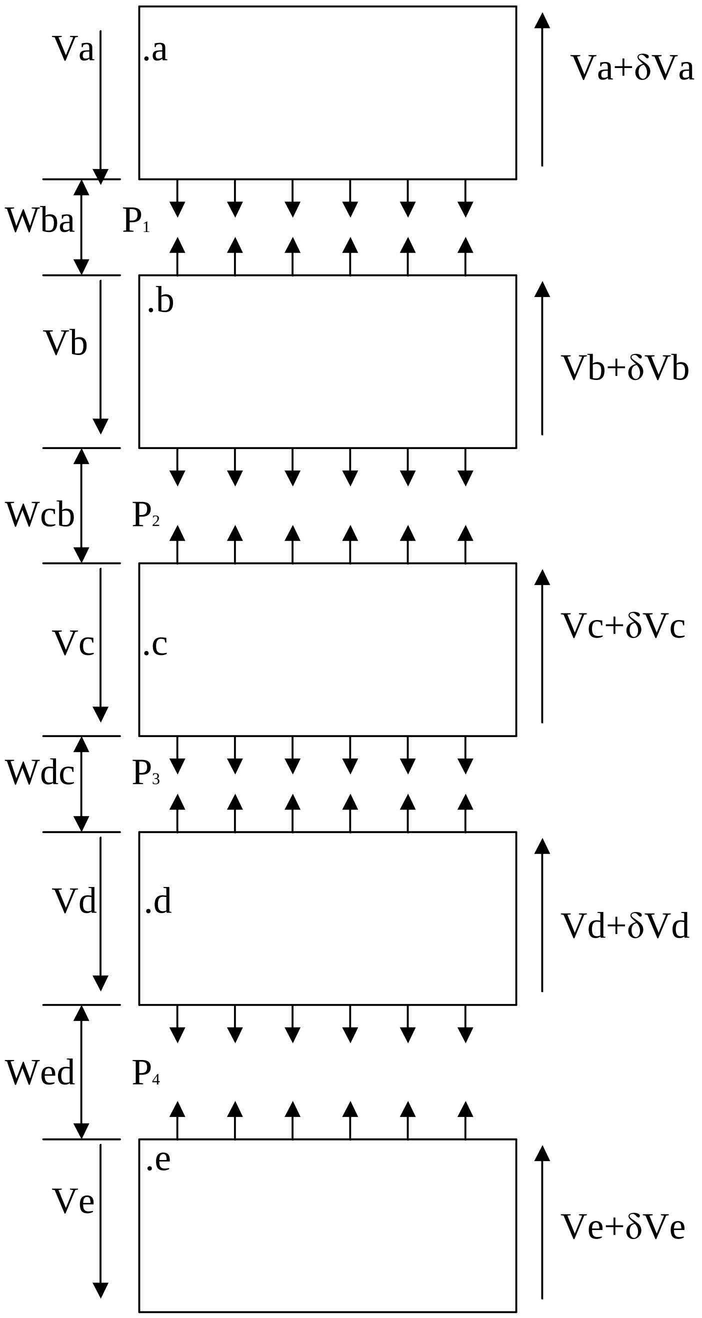


Figure (3-c) Composite five layers in separation

Figure (3) Composite five layers element beam.

## Conclusion

Composite multi-layered beams is relatively new techniques used in many engineering fields, specially marine construction for the major benefits provided by such structures. A derivations of three, four and five layers composite simply supported beams based on Roberts' approach led to set of governing partial differential equations, using equilibrium and compatibility conditions, which can be solved by finite difference method with a proper boundary conditions , No. of these equations depending on D.O.F in each layer. General formula was derived to obtained the governing equations for and layer composite simply supported beam under uniform loading.

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NOTATION

a, b, and c= Subscript denotes different layers.

$A_a$, $A_b$ and $A_c$ = Cross-sectional area of different layers.

$A$ = Effective width of concrete slab.

$d_1$ and $d_2$ =Distance between the centroids of successive layers.

$E_1$ = Modulus of elasticity of concrete.

$E_2$ = Modulus of elasticity of steel.

$E_a$, $E_b$ and $E_c$ =Modulus of elasticity of different layers .

$F_a$, $F_b$ and $F_c$ =The axial forces in different layers.

$h_a$, $h_b$ and $h_c$ = Thickness of different layers.

$I_a$, $I_b$ and $I_c$ =Second moment of area for the layer a.

$I_1$ and $I_2$ = Moment of inertia of concrete slab and steel about its own centroid.

$k_{s1}$ and $k_{s2}$ =Shear stiffness of the joint per unit length between successive layers.

$k_{n1}$ and $k_{n2}$ =Normal stiffness of the joint per unit length between successive layers.

L = span length.

M= External applied moment.

$M_a$, $M_b$ and $M_c$ =Moment for layer a.

$P_1$ and $P_2$ =Normal force per unit length at the upper and lower interface.

$r_i$ =Live load.

$r$ =Live load and dead load.

$r_a$, $r_b$ and $r_c$ = Distributed self-weight of layer a.

$R_r$, $R_l$ =Reaction at the right and the left supports.

$U_{ab}$ and $U_{bc}$ = Slip between upper and lower layers.

$u_a$, $u_b$ and $u_c$ =Displacements of the different layers in the x -direction.

W= Point load.

$w_a$, $w_b$ and $w_c$ =Displacements of the layer a, b and c in the z -direction.

$w_{ba}$, $w_{cb}$ =Separation at the interface between the upper and lower layers.

x.= Subscript denote differentiation.

$z_{ai}$, $z_{ai}$ and $z_{ai}$ =Z-coordinate of interface relative to local x-z axes in layers a, b and c.

$e_f$ =Free strain due to shrinkage, temperature etc.

$e_r$ = Strain induced during the construction sequence.

$\bar{e}$ =Integration of strain function over cross section area of the material.

$e_a$, $e_b$ and $e_c$ =Strain in layers a , b and c.

$s_a$, $s_b$ and $s_c$ =Stress in layers a, b and c.

$\Delta x$ =Spacing between nodes.