Optimum Design of Singly and Doubly Reinforced Concrete Rectangular Beam Sections: Artificial Neural Networks Application

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Abstract

Construction of concrete structures involves at least two different main materials: concrete and steel. Design of these structures should be based on cost rather than weight minimization. In this work, least cost design of singly and doubly reinforced beams is done by applying of the Lagrangian multipliers method (LMM) under ultimate design constraint beside other constraints. Cost objective functions and moment constraints are derived and implemented within the optimization method. The optimum solution comparisons with conventional design methods are performed and the result reported, showing that the LMM can be successfully applied to the minimum cost deign of reinforced concrete beams without need for iterative trials. Optimum design solution surfaces have been developed. Good and reliable results have been obtained and confirmed by using standard design procedures. The artificial neural networks (ANN) has been trained with design data obtained from optimal design formulas. After successful trials, the model predicted the optimum depth of the beam sections and optimum areas of steel required for the problems with accuracy satisfying all design constraints.

التصميم الأمثل لمقاطع العتبات الخرسانية المسلحة المستطيلة المفردة والمزدوجة التسليح : تطبيق الشبكات العصبية الاصطناعية

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الخلاصة

يتم إنشاء المنشآت الخرسانية باستخدام مادتين مختلفتين على الأقل هما الحديد والخرسانة وتصميم مثل هذه المنشآت يعتمد على التقليل من الكلفة أكثر من تقليل الوزن. في هذه الدراسة تم تطبيق طريقة (لاكرانج) في تصميم المقاطع الخرسانية مفردة ومزدوجة التسليح بأقل كلفة حسب متطلبات طريقة التصميم القصوى. إن دالة كلفة الهدف و تقييد العزم تم اشتقاقها وتطبيقها ضمن طريقة التقليل المثالية. نتائج التحليل المثالية تمت مقارنتها مع طرق التصميم التقليدية وتمت جدولة النتائج ، التي تبين منها أن طريقة (لاكرانج) يمكن أن تطبق بنجاح لتصميم المقاطع الخرسانية دون الحاجة إلى محاولات تكرارية بالإضافة إلى أنه تم رسم سطح التصميم الأمثل للمقطع الخرساني المسلح . النتائج كانت جيدة و مقبولة و موثوقة باستخدام الطرق القياسية. وقد تم تطبيق الشبكات العصبية الاصطناعية مع البيانات التي تم الحصول عليها من معادلات التصميم الأمثل. بعد النجاح في التطبيق ، تنبأ النموذج بعمق مقطع العتبة ومساحة حديد التسليح اللازمة للمقطع مع دقة في التصميم لتلبية جميع القيود.

1. Introduction

Structural design is an iterative process. The initial design is the first step in design process. Though the various aspects of structural design are controlled by many codes and regulations, the structural engineer has to exercise caution and use his judgment in addition to calculations in the interpretation of the various provisions of the code to obtain an efficient and economic design. After the design process, the designer makes an overall guess about the possible optimum solution consistent with designer's experience, knowledge, constraints, and requirements. The analysis of the structure is then carried out using initial design. Based on the results of the analysis a re-design of the structure is carried out if any of the constraints is not satisfied. The efficiency of the design process depends heavily on initial guess. A good initial design reduces the number of subsequent analysis–design cycles. This phase is extremely difficult to computerize as it needs human intuition. In recent years efforts have been made to computerize the initial design process using artificial neural networks as they can learn from available designs during training process.

Optimization of building structures is a prime target for designers and has been investigated by many researchers in the past (Tam Ha [1], Rath *et al.* [2], Ceranic, and Fryer, [3] , Jarmai *et al.* [4], Matej and Michal [5], Barros, *et al.* [6], Sahab *et al.* [7], Zou *et al*. [8] and Aschheim *et al*. [9]).

Optimization is highly linked to the selection of the most suitable structural system. Such a system would still be sized to ensure the least overall cost. In structural design, many parameters are incremental in their nature rendering a continuous approach almost impossible to implement in a practical optimization exercise.

Artificial neural network is a new technology emerged from approximate simulation of human brain and has been successfully applied in many fields of engineering. Neural networks demonstrate powerful problem solving ability. They are based on quite simple principles but take advantage of their mathematical nature in terms of non-linear iteration. Neural networks with Back Propagation (BP) learning showed results by searching for various kinds of functions. However, the choice of basic parameters (Network topology, learning rate, initial weights) often already determines the success of the training process. However, there are no clear rules how to set these parameters. Yet these parameters determine the efficiency of training. Lot of research has taken place on applications of artificial neural networks in structural engineering. Artificial Neural Networks ANNs have been used in the fields of concrete structures for nearly 25 years. The main results were achieved in the structural design process and the structural analysis, for instance, Tang *et al.* [10]; Oreta [11], Fonseca *et al.* [12], D. Maity and A. Saha [13]. The ANN models built by these researchers basically set the structural parameters such as the material property, the boundary condition and the size of a structure as the input of the ANN model to predict the ability for the structure to resist the load. In most of these works the neural networks have been trained by using back propagation algorithm. In this approach the connection weights of neural networks are initially set to some random values. These values are then modified automatically according to the learning algorithm during the process of learning.

In this work, the optimal design information has been incorporated into an artificial neural network (ANN) which gives optimal design, satisfying all of the criteria in one step. The optimization involves choosing of the design variables in such a way that the cost of the beam is the minimum, subject to the satisfaction of behavioural and geometrical constraints as per recommended method of design codes.

2. Structural Optimization

In optimization problems the aim is to minimize the weight, volume or the cost of the structure under certain deterministic behavioural constraints. The mathematical formulation of typical structural optimization problem with respect to the design variables, the objective and constraint functions can be expressed in standard mathematical terms as a non-linear programming problem as follows [14]

Min $F(s)$

subjected to

$$h_j(s) \le 0,\ j=,1 \ ....m$$
$$s_i^{\,l} \le s_i \le s_i^{\,u},\ i=1......n \tag{1}$$

where s is the vector of design variables, $F(s)$ the objective function to be minimized, $h_j(s)$ the behavioural constraint, $s_i^{\,l}$ and $s_i^{\,u}$ are the lower and the upper bounds of typical design variable s_i.

The set of design variables gives a unique definition of a particular design. The selection of design variables is very important in the optimization process. The designer has to decide a priori where to allow design changes to evaluate how these changes should take place by defining the location of the design variables and the moving directions.

2.1 Lagrange Multipliers Method

In its original formulation, the LMM applies to the optimization of a multivariate objective function expressed as

$$y = f(x_1, x_2, ..., x_n), \tag{2}$$

subjected to the equality constraints of the form

$$g_i(x_1, x_2, ...x_n) = 0, \qquad i = 1,2, ..., m \tag{3}$$

where n is the number of independent variables and m are the number of constraints; m must be less than n by definition of the problem. The procedure is to construct the unconstrained Lagrangian function L of the form

$$L = (x_1, x_2, ..., x_n, \lambda_1, \lambda_2, ..., \lambda_m) = f(x_1, x_2, ...x_n) + \sum_{i=1}^{m} \lambda_i g_i(x_1, x_2, ..., x_n), \tag{4}$$

where the unspecified constraints λ_i are the Lagrange multipliers determined in the course of the extremization. The necessary conditions for L to possess an extreme (stationary point) are

$$\frac{\partial L}{\partial x_k} = \frac{\partial f}{\partial x_k} + \sum_{i=1}^{m} \lambda_i \frac{\partial g_i}{\partial k_i} = 0, \quad k = 1,2,.....n, \tag{5}$$

$$\frac{\partial L}{\partial \lambda_i} = g_i = 0, \quad i = 1,2,.....,m. \tag{6}$$

Expression (6) simply restates the original constraints acting on the solution space of the objective function $y = f(x_1, x_2, ..., x_n)$. Expressions (5) and (6) are a system of $n+m$ equalities with $n+m$ unknowns. Hence, their solution will yield stationary values for $x_1, x_2, ...x_n$ and $\lambda_1, \lambda_2, ..., \lambda_m$ from which the optimum solution can be obtained.

3. Singly Reinforced Beam Section (SRB)

3.1 Problem Formulation

Figure (1) shows a typical single reinforced rectangular section with simplified rectangular stress block. The following factors are defined and are assumed fixed for a given problem:

$$t = \frac{d_s}{d} \tag{7}$$

In Eq.(7), t (which is the geometrical property) is a function of the effective depth, d , to be determined. Therefore, this factor is variable. Since the range of values of t is generally limited and its influence on total cost of the beam section is small, it is satisfactory to assume t to be constant.

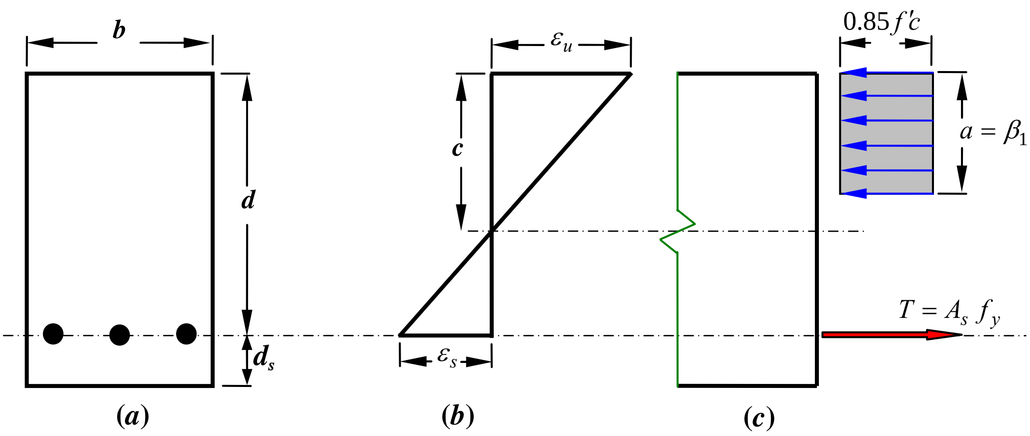

Fig. 1. Singly reinforced rectangular beam

When a rectangular-beam section is designed, the nominal bending moment, M_n, with cross section width b, and material properties $f'c$ and f_y are generally given. Thus, d and A_s are to be determined. In this formulation however, R, and ρ in Eqs.(8 and 9), which follows, are used as design variables of the optimum design problem instead of d and A_s,

$$d = R\sqrt{\frac{M_n}{b}} \quad (8)$$

$$A_s = \rho \; b.d \quad (9)$$

where R is a coefficient used to determine effective depth which is calculate from optimum solution later. A cost function is defined as the total cost (C) which is equal to costs of flexural reinforcement plus concrete. These costs involve material costs and fabrication costs, respectively. Let C_s and C_c refer to the unit costs of steel and concrete for a unit volume. The cost of the beam of unit length is:

$$C = C_s.V_s + C_c.V_c \quad (10)$$

where V_s and V_c are volumes of steel and concrete per unit length of beam, respectively. Eq.(10) can be written as:

$$V_s = 1 \times A_s = \rho \; b.d \quad (11)$$

$$V_c = 1 \times [(d + d_s)b] = [(1+t)b.d] \quad (12)$$

Substituting Eqs.(11 and 12) in Eq.(10) yields:

$$C = [\rho q + (1+t)] \; R.C_c \sqrt{M_n.b} \quad (13)$$

in which ($q = C_s / C_c$) is a ratio of the unit cost of steel to that of concrete. As ($C_c \sqrt{M_n.b}$) in Eq.(13) is constant for a given problem, then minimizing the cost function (C) is equivalent to minimizing

$$w_{(\rho,R)} = [\rho q + (1+t)] \; R \quad (14)$$

The constrain function:

Geometry of the rectangular beam is shown in Fig.(1) together with the simplified rectangular stress block as given in the ACI-Code [15]:

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) \quad (15)$$

in which:

$$a = \frac{A_s\ f_y}{0.85\, f'c\,.b} \tag{16}$$

$$M_u = \phi M_n \tag{17}$$

$$\rho_1 \le \rho \le \rho_u \tag{18}$$

$$\rho_1 = \frac{1.4}{f_y} \quad or \quad \frac{0.25\sqrt{f'c}}{f_y} \tag{19}$$

$$\rho_u = 0.85\,\beta_1 . \frac{f'_c}{f_y} . \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t} \tag{20}$$

The factor β_1 in Eq.(20) shall be taken as 0.85 for concrete strength $f'c$ up to and including 28 MPa. For strengths above 28 MPa, β_1 shall be reduced continuously at a rate of 0.05 for each 6.9 MPa of strength in excess of 28 MPa, but β_1 shall not be taken less than 0.65.

To ensure under reinforced behavior, ACI Code; sec.10.3.5 establishes a minimum net tensile strain ε_t of 0.004 at the nominal member strength for members subjected to axial loads less than $0.1 f'_c A_g$, where A_g is the gross area of the cross section.

The ACI Code further encourages the use of lower reinforcement ratios by allowing higher strength reduction factors in such beams. The Code defines a tension-controlled member as one with a net tensile strain greater than or equal to 0.005. The corresponding strength reduction factor is $\phi = 0.9$. The Code additionally defines a compression-controlled member as having a net tensile strain of less than fy / E_S. The strength reduction factor ϕ for compression-controlled members is 0.65. A value of $\varepsilon_t = fy / E_S$ is a yield strain for steel. Between net tensile strains of fy / E_S and 0.005, the strength reduction factor varies linearly, and the ACI Code allows a linear interpolation of ϕ based on ε_t, as shown in Fig.(2). Calculation of the nominal moment capacity frequently involves determination of the depth of the equivalent rectangular stress block *a*. Since $c = a / \beta_1$, it is some times more convenient to compute c/d ratios than the net tensile strain.

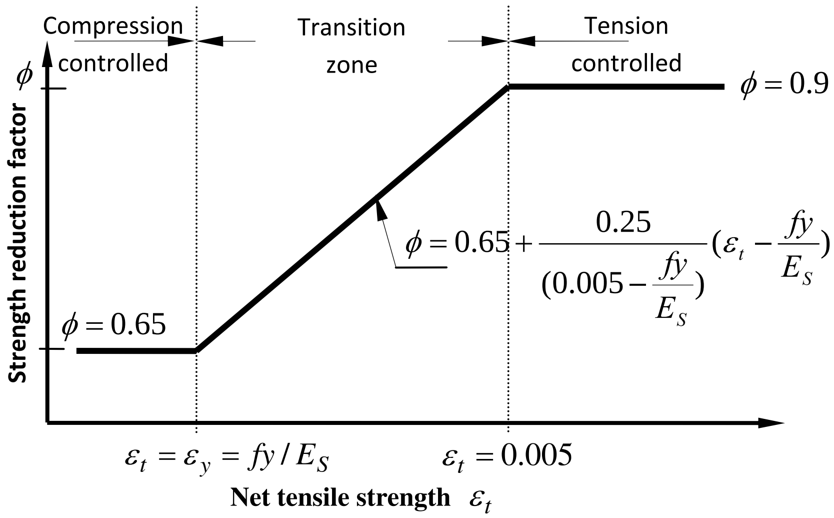

Fig. 2. Variation of strength reduction factor ϕ with net tensile strain ε_t [16].

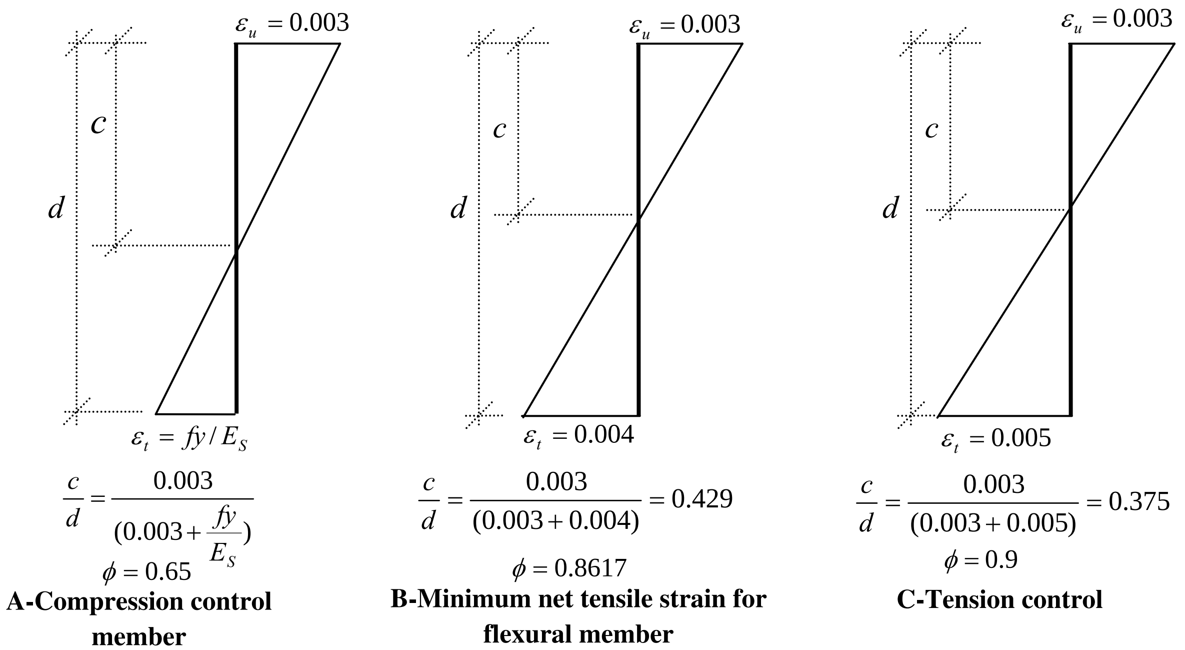

Fig. 3. The net tensile strain ε_t and c/d ratios for singly reinforced concrete beam [16].

The assumption that plane sections remain plane ensures a direct correlation between net tensile strain and the c/d ratio, as shown in Fig.(3). In accordance with the safety provisions of the ACI Code, the net tensile strain is checked, and if $\varepsilon_t \geq 0.005$, this nominal capacity is reduced by the factor $\phi = 0.9$ to obtain the design strength. For ε_t between fy / E_S and 0.005, ϕ must be adjusted, as discussed earlier.

Substituting Eqs.(7, 8, 9, and 16) into Eq.(15), obtain:

$$\rho f_y \left(1 - \frac{\rho f_y}{1.7 f'_c} \right) R^2 - 1 = 0 = g(\rho, R) \tag{21}$$

Thus, the optimum design problem is to minimize $w_{(\rho,R)} = [\rho q + (1+t)] R$ subjected to the constraints:

$$\rho_1 \leq \rho \leq \rho_u \quad \text{and} \quad \rho f_y \left(1 - \frac{\rho f_y}{1.7 f'_c} \right) R^2 - 1 = 0 \tag{22}$$

3.2 Optimization and Procedure of Calculations

The LMM (Lagrangian Multipliers method) applies to the optimization of a multivariate objective function expressed as[14]:

$$L(\rho, R, \lambda) = w_{(\rho,R)} - \lambda [g(\rho, R)] \tag{23}$$

$$L(\rho, R, \lambda) = [\rho q + (1+t)] R - \lambda \left[\rho f_y \left(1 - \frac{\rho f_y}{1.7 f'_c} \right) R^2 - 1 \right] \tag{24}$$

where the unspecified parameter λ is the Lagrangian Multipliers. Three independent variables ρ, R and λ appear in the cost objective function, Eq.(23). Derivatives with respect to the three independent variables; produce three equations as given below:

$$q - \lambda \left[f_y \left(1 - \frac{\rho f_y}{0.85 f'_c} \right) \right] R = 0 \tag{25}$$

$$\left[\rho\, q + (1+t)\right] - \lambda\left[\rho\, f_y\left(1 - \frac{\rho\, f_y}{1.7\, f_c'}\right)2R\right] = 0 \tag{26}$$

$$\rho\, f_y\left(1 - \frac{\rho\, f_y}{1.7\, f_c'}\right)R^2 = 1 \tag{27}$$

By eliminating λ from Eq.(25) and Eq.(26), ρ_{opt}^{m} is obtained as:

$$\rho_{opt}^{m} = 1\Big/\left(\frac{q}{1+t} + \frac{f_y}{0.85\, f'c}\right) \tag{28}$$

and using Eq.(27) , R_{opt}^{m} is obtained as:

$$R_{opt}^{m} = 1\Big/\sqrt{\rho_{opt}\, f_y (1 - \frac{\rho_{opt}\, f_y}{1.7\, f'c})} \tag{29}$$

Taking Eq.(19) and Eq.(20) into consideration, the optimum steel ratio ρ_{opt}, and optimum coefficient R_{opt}, are given as:

$$\left.\begin{array}{lll} \rho_{opt} = \rho_{opt}^{m} & ; \; R_{opt} = R_{opt}^{m} & if\ \rho_1 < \rho_{opt}^{m} < \rho_u \\ \rho_{opt} = \rho_1 & ; \; R_{opt} = R_u & if\ \rho_{opt}^{m} \le \rho_1 \\ \rho_{opt} = \rho_u & ; \; R_{opt} = R_1 & if\ \rho_{opt}^{m} \ge \rho_u \end{array}\right\} \tag{30}$$

Values of R_u and R_1 are found as follows:

$$R_u = 1\Big/\sqrt{\rho_1\, f_y (1 - \frac{\rho_1\, f_y}{1.7\, f'c})} \quad ; \quad R_1 = 1\Big/\sqrt{\rho_u\, f_y (1 - \frac{\rho_u\, f_y}{1.7\, f'c})} \tag{31}$$

By referring to Eqs.(8 and 9) the optimum effective depth, d_{opt}, and the optimum area of steel As_{opt}, are:

$$d_{opt} = R_{opt}\sqrt{\frac{(M_u / \phi)}{b}} \quad ; As_{opt} = \rho_{opt}.b.d_{opt} \tag{32}$$

4. Doubly Reinforced Beam Section (DRB)

4.1 Problem Formulation

Based on the similarity with the total cost function per unit length for the doubly reinforced rectangular section shown in Fig.(4) may be written as Eq.(13) as:

$$C = \left[\left(\rho_{doubly} + \rho'\right)q + (1+t)\right]R.\ C_c\sqrt{M_n\, b} \tag{33}$$

The ACI Code limits the net tensile strain, not the reinforcement ratio. To provide the same margin against brittle failure as for singly reinforced beams, the area of reinforcement should be limited

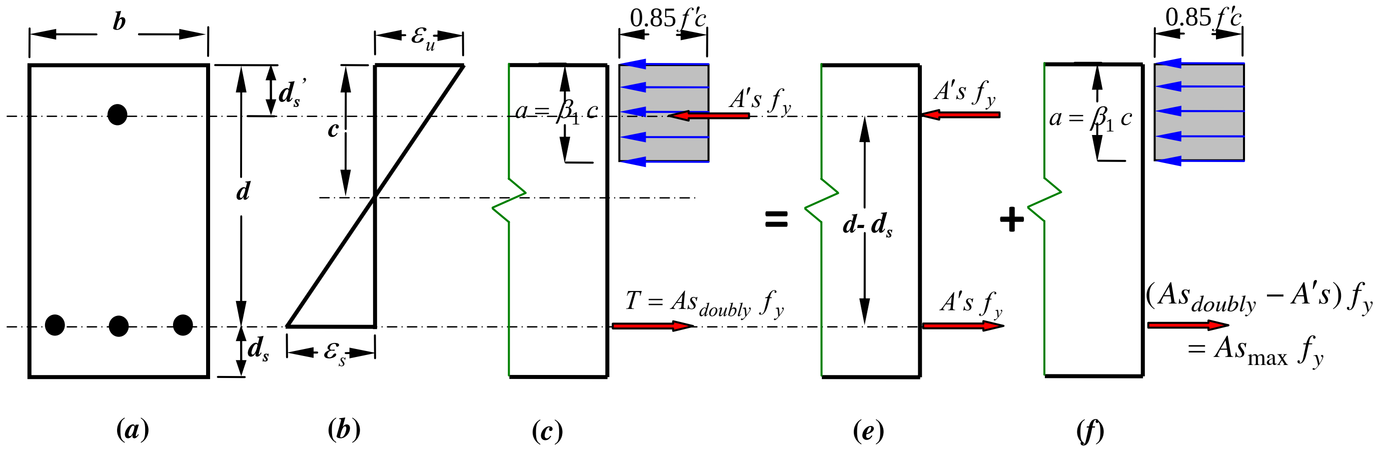

Fig. 4. Bending stress and strain distribution in cross-section of doubly reinforced rectangular beam.

To; $As_{doubly} - A's = As_{(\max.)}$ as shown in Fig.(4.*f*). It is easily shown that the reinforcement ratio ρ_{doubly} for a doubly reinforced beam is [16]:

$$\rho_{doubly} = \rho_u + \rho' \tag{34}$$

where ρ_u is the maximum reinforcement ratio allowed by the ACI Code for singly reinforcement beams and given by Eq.(20).

As ρ_u establishes location of the neutral axis, the limitation of Eq.(34) will provide acceptable net tensile strains. A check of ε_t is required to determine the strength reduction factor ϕ and verify that the net tensile strain requirements are satisfied. In the case of $\varepsilon_t \geq 0.005$, ρ_u may be replaced by ρ in Eq.(34) which gives $\phi = 0.9$.

Substituting Eq.(34) into Eq.(33), produces the following cost function, *C*:

$$C = [(\rho_u + 2\rho')q + (1+t)]\ R\ C_c \sqrt{M_n.b} \tag{35}$$

Since the product $C_c \sqrt{M_n.b}$ in Eq.(35) is constant for a given problem, minimization of the cost function *C* is equivalent to minimizing

$$w_{(\rho',R)} = [(\rho_u + 2\rho')q + (1+t)]\ R \tag{36}$$

The constraint function:

Fig.(4), shows the geometry and the simplified rectangular stress block for the cross- section of rectangular of rectangular doubly reinforced beam. When the ultimate design moment M_u exceeds the moment of resistance of a singly reinforced section ($k_n b d^2$), compression reinforcement is required, Considering equilibrium of the horizontal forces on the beam cross- section for this case, depth of the rectangular compression block a is equal to ($\frac{(As_{doubly} - A's) f_y}{0.85 f'c\, b}$). Using Eq.(34) ; the block depth will then be equal to:

$$a = \frac{\rho_u f_y}{0.85 f'c}.d \tag{37}$$

The ACI-Code [15] specifies requirements for M_n and ρ for a doubly reinforced concrete beam section (taking moments about the tension reinforcement) as [16]:

$$M_n = (As_{doubly} - A's) f_y (d - \frac{a}{2}) + A's f_y (d - d_s) \tag{38}$$

and:

$$\rho'_{cy} \le \rho_{doubly} \le \rho'_{\max} \tag{39}$$

where ρ'_{cy} gives minimum tensile reinforcement ratio that will ensure yielding of the compression steel at failure [16]:

$$\rho'_{cy} = 0.85\,\beta_1 . \frac{f'_c}{f_y} . \frac{d'_s}{d} \; \frac{\varepsilon_u}{\varepsilon_u - \varepsilon_y} + \rho' \tag{40}$$

$$\rho'_{\max} = \rho_u + \rho' \tag{41}$$

Substituting Eqs.(7),(8),(9),(34) and Eq.(37) into Eq.(38), yields:

$$\left[\rho_u \,(1 - \frac{\rho_u\, f_y}{1.7\, f'c}) + \rho'(1-t) \right] R^2 = \frac{1}{f_y} \tag{42}$$

Thus, the optimum design problem is to minimize Eq.(36) which is subjected to the constraints:

$$\rho'_{cy} \le \rho_{doubly} \le \rho'_{\max} \,,\; g(\rho', R) = \left[\rho_u \,(1 - \frac{\rho_u\, f_y}{1.7\, f'c}) + \rho'(1-t) \right] R^2 - \frac{1}{f_y} = 0 \tag{43}$$

4.2 Optimization and Procedure of Calculations

By excluding Eq.(39), the constraint on the problem is given by Eq.(42). Then using the LMM, technique [14], Eq.(43) can be solved leading to a set of design variables. Accordingly a Lagrangian function *L*, is defined as:

$$L(\rho', R, \lambda) = \left[(\rho_u + 2\rho')q + Q \right] R - \lambda \left[\left[\rho_u \,(1 - \frac{\rho_u\, f_y}{1.7\, f'c}) + \rho'(1-t) \right] R^2 - \frac{1}{f_y} \right] \tag{44}$$

in which $Q = 1 + t$.

Setting $\partial L/\partial \rho' = 0$, $\partial L/\partial R = 0$, $\partial L/\partial \lambda = 0$, yields

$$2q - \lambda \left[(1-t)\, R \right] = 0 \tag{45}$$

$$\left[(\rho_u + 2\rho')q + Q \right] - \lambda \left[\left[\rho_u \,(1 - \frac{\rho_u\, f_y}{1.7\, f'c}) + \rho'(1-t) \right] 2R \right] = 0 \tag{46}$$

$$\left[\rho_u \,(1 - \frac{\rho_u\, f_y}{1.7\, f'c}) + \rho'(1-t) \right] R^2 = \frac{1}{f_y} \tag{47}$$

By eliminating λ from Eq.(45) and Eq.(46), ρ'_{opt}, R_{opt} , are obtained as:

$$\rho'_{opt} = \frac{\rho_u\, q \left(\frac{\rho_u\, f_y}{0.425\, f'c} - (3+t) \right) + (1-t)\; Q}{2q\; (1-t)} \tag{48}$$

$$R_{opt} = 1 \Big/ \sqrt{f_y \left[\rho_u \,(1 - \frac{\rho_u\, f_y}{1.7\, f'c}) + \rho'_{opt}\,(1-t) \right]} \tag{49}$$

The optimum effective depth d_{opt} for (DRB), the optimum area of steel in tension As_{opt}, and the compression steel area $A's_{opt}$ are obtained as:

$$d_{opt} = R_{opt} \sqrt{\frac{(M_n / \phi)}{b}} \tag{50 a}$$

$$As_{opt} = (\rho_u + \rho'_{opt}).b.d_{opt} \tag{50 b}$$

$$A's_{opt} = \rho'_{opt}\, b.d_{opt} \qquad (50\ c)$$

The procedure to find the optimum solution (*i.e.* $d_{opt}, As_{opt}, A's_{opt}$) is summarized in numerical design examples.

5. Numerical Examples

Three typical design examples are given, illustrating situations where the optimum solution is either a singly or doubly reinforced section. For given values of $q, t, f_y, f'c$, the optimum solution is obtained and presented graphically. The optimum solution is compared with the standard design procedure specified in ACI-Code [15].

5.1 Design Example 1: Singly Reinforced Beam (SRB)

A rectangular beam section with *b*=300 *mm* is given. It is required to determine values of the optimum area of steel As_{opt} and the optimum effective depth d_{opt}, for M_u=667 *kN.m*, $f'c$=28 *MPa* and f_y=414 *MPa*. It is assumed that *t*=0.1, and *q*=85.

Figure (5), shows the optimum solution for singly reinforced concrete beam section (SRB). Hence, from Eq.(28) ρ_{opt} is 0.010563270 giving the corresponding optimum coefficient of the effective depth of the section R_{opt} obtained from Eq.(29) as 0.5017965. The optimum area of the tension reinforcement As_{opt} and optimum effective depth of the section d_{opt} are then obtained from Eq.(32) as 2499 mm^2 and 788.7 mm respectively.

On the bases of the depth wise strain variation shown in Fig. (3), value of the net tensile strain is ε_t $0.01088 > 0.005$, so the strength reduction factor is $\phi = 0.9$. The corresponding total material cost C of the beam per unit length is then obtained from Eq.(13) to be 0. 4727144 C_c \$/m as its minimum value (in terms of the concrete cost per unit volume). Figure (5) shows also that the optimum solution lies on the bending moment constraint boundary with the cost objective function being tangential to the curve. Table 1 shows the results using the standard design method. It is marked from this table that the derived optimum design formulae for singly reinforced sections gives an accurate estimate of the minimum material cost.

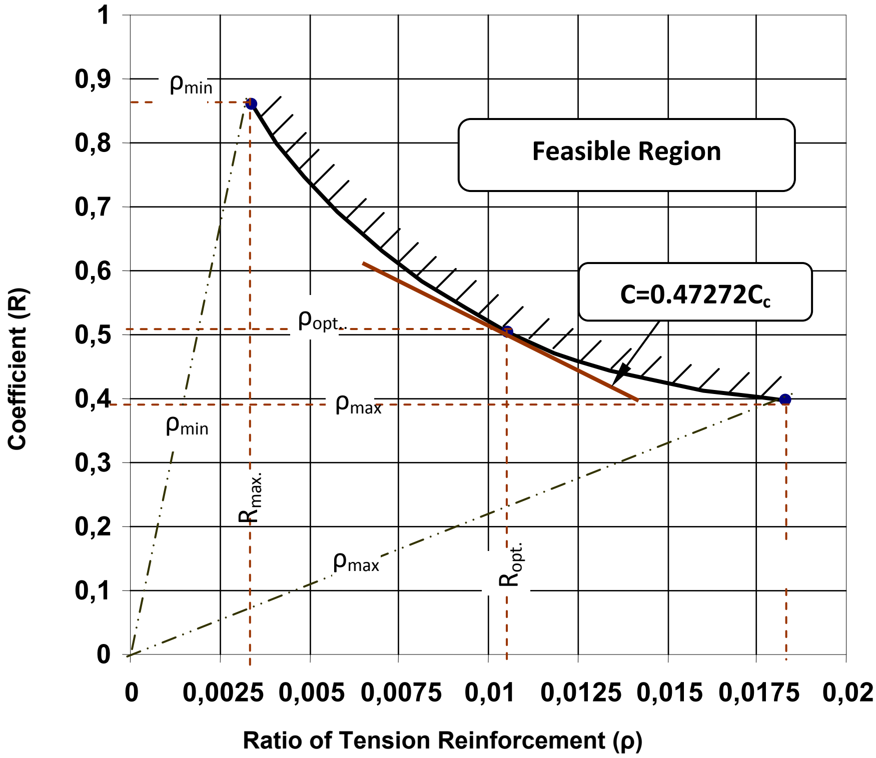

Fig. 5. Optimum design for the singly reinforced concrete beam of Example 1.

Table 1. Results of the standard design method and LMM for the singly reinforced beam of Example 1.

Effective depth (d) mm	Area of tension Reinforcement (As) mm^2	Tension Reinforcement ratio (ρ)	Total material costs (in terms of C_c) (\$/m)
622.4*	3421	0.018324276**	0.4962165
660	3147.4710	0.015896318	0.4853351
700	2907.4170	0.013844842	0.4781304
740	2705.9530	0.012188977	0.4742060
788.7	**2499**	**0.010563270**	**0. 4727144**
780	2533.6260	0.010827461	0.4727582
820	2384.0240	0.009691154	0.4732420
860	2252.596	0.008730992	0.4752706
900	2135.997	0.007911100	0.4785597

*minimum value of *the effective depth* which is calculated from the minimum coefficient R_1 using Eq.(31).

**maximum reinforcement ratio, given by Eq.(20).

5.2 Design Example 2: Doubly Reinforced Beam (DRB)

A-rectangular reinforced concrete beam section with b=250 mm, $f'c = 20\,MPa$ and $f_y = 400\,MPa$; is given. It is required to determine values of the optimum effective depth d_{opt} and optimum area of steel As_{opt} in which M_u =497 *kN.m*. Assume values of t and q as 0.1 and 20, respectively.

The optimum result is presented graphically on the design surface (ρ, R) of Fig.(6). Using Eq.(48), value of ρ'_{opt} is obtained to be 0.008967 giving a corresponding value for ρ_{opt} as 0.022514 . Value of R_{opt} is obtained from Eq.(49) as 0.3584414. value of the optimum area of the tension reinforcement As_{opt} is calculated from Eq.(50-b) to be 2998.456 mm^2, while value of $A's_{opt}$ is obtained by applying Eq.(50-c) as 1194.258 mm^2 after computing value of the optimum effective depth of the section d_{opt} from Eq.(50-a) as 532.73 mm.

According to the strain variation in the depth wise direction shown in fig.(3), value of the net tensile strain ε_t is $0.0064 > 0.005$, so the strength reduction factor ϕ value is 0.9, then the total material cost C of the beam per unit length is obtained from Eq.(35) to be $0.230354\,C_c$ $\$/m$ at its minimum value. The optimum solution lies on the tangent point of doubly reinforced bending constraint moment with the objective function being tangential to the curve.

Table 2 shows the results of the standard design method including values of the effective depth, area and ratio of the tension reinforcement and the total cost of the beam per unit length in terms of concrete cost C_c per unit volume. The row of the optimum is the shaded one.

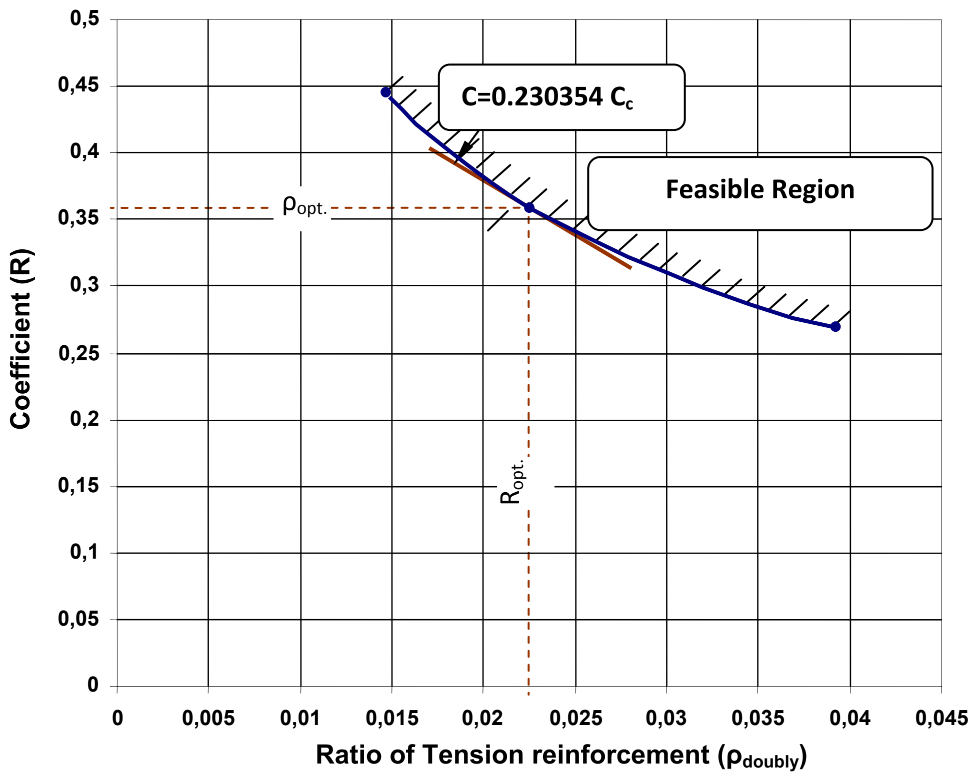

Fig. 6. Optimum design for the doubly reinforced concrete beam of Example 2.

Table 2. Results of the standard design method and LMM for the doubly reinforced beam of Example 2.

Effective depth (d)mm	Area of tension Reinforcement (As) mm^2	Tension Reinforcement ratio (ρ_{doubly})	Area of compression Reinforcement ($A\acute{}s$) mm^2	compression Reinforcement ratio (ρ')	Total material costs (in terms of C_c) (\$/m)
400	3924	0.03924	2570	0.025696	0.2398762
440	3585	0.03259	2094	0.019040	0.2345793
470	3369	0.02867	1777	0.015123	0.2321643
500	3180	0.02544	1486	0.011890	0.227814
520	3066	0.02358	1305	0.010038	0.227298
532.73	**2998.456**	**0.022514**	**1194.258**	**0.008967**	**0.230354**
540	2961	0.02193	1132	0.008389	0.2303755
560	2864	0.02045	968	0.006913	0.2306415
590	2732	0.01852	734	0.004973	0.2315563
620	2613	0.01686	513	0.003309	0.2330103
640	2540	0.01588	372	0.002327	0.2342417
660	2472	0.01498	236	0.001433	0.2356606

5.3 Design example 3: (SRB-DRB)

Given quantities are the same as those of Example No.2, except that q=30, The results are as follows:

$\rho^{m}_{opt} = 0.01968 > \rho_u$;hence $\rho_{opt} = \rho_u = 0.0135469$,depth= d_1 =696.364 mm, As_{opt} =2358.389 mm^2.

In this example, the optimum section agrees with the section using ρ_u as the steel ratio. The corresponding value of the total material cost C of the beam per unit length is then obtained from Eq.(13) to be $0.2622517 C_c$ $\$ / m$ at its minimum limit (in terms of concrete cost per unit volume). Fig.(7) shows the optimum result is presented graphically on the 2D-design surface (ρ, R). The design space is discontinuous with the feasible region consisting of a singly (SRB) and doubly (DRB) reinforced solution space. The comparison between the standard design method and the optimum solution is also summarized in Table 3. The optimum solution lies on the bending moment constraint boundary at the point of intersection with the boundary reinforcement, as shown in Fig.(7). As in the previous example the cost objective function is tangential to the bending moment constraint surface.

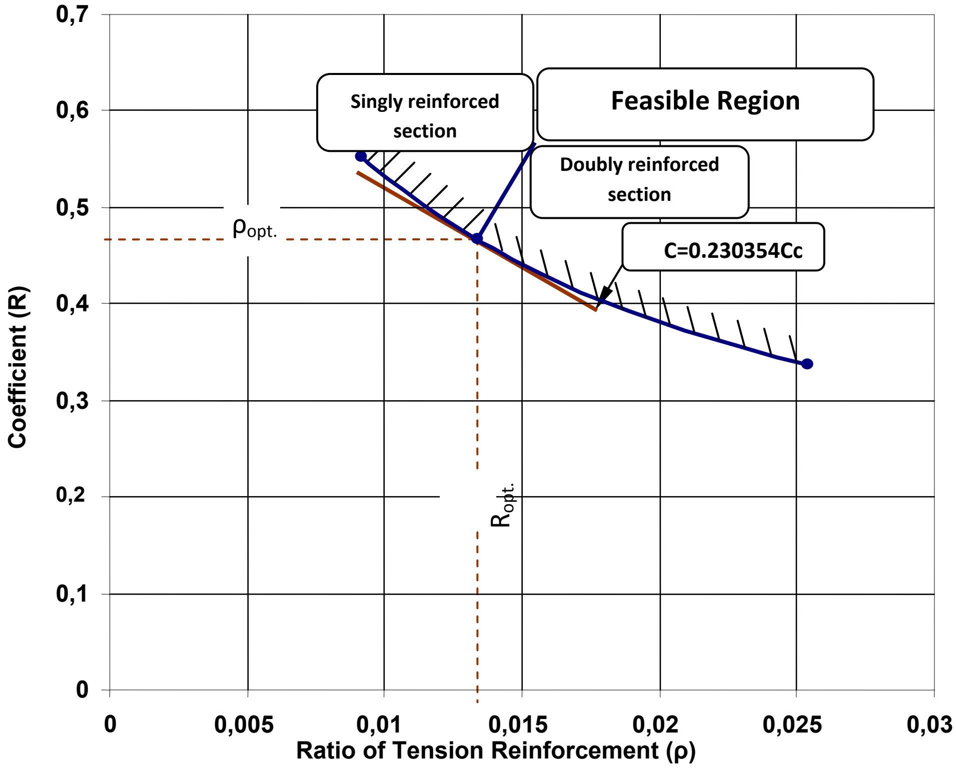

Fig. 7. Optimum design for the SRB-DRB reinforced concrete beam of Example 3.

Table 3. Results of the standard design method and LMM for the SRB-DRB reinforced beam of Example 3.

Effective depth (d)mm	Area of tension Reinforcement (As) mm^2	Tension Reinforcement ratio (ρ)	Area of compression Reinforcement (A'_s) mm^2	compression Reinforcement ratio (ρ')	Total material costs (in terms of C_c) (\$/m)
500	3180	0.0254400	1486	0.0118900	0.2774762
540	2961	0.0219333	1132	0.0083887	0.2713132
570	2818	0.1977544	888	0.0062320	0.2679469
600	2691	0.0179400	659	0.0043907	0.2654776
635	2558	0.0161134	407	0.0025637	0.2635609
696.364	**2358.389**	**0.0135469**	**-**	**-**	**0.2622517**
750	2124	0.0113266	-	-	0.2699621
780	2015	0.0103327	-	-	0.2749462
800	1949	0.0097459	-	-	0.2784755
820	1888	0.0092108	-	-	0.2821465

6. Neural Network Approach

6.1 Neural Network Design and Training

The developed database for the optimum design of rectangular sections, which is based on the equations in articles 3 and 4, were used to train a neural network. The design input to the problem includes: applied moment, M_u, concrete strength fc, yield strength of steel reinforcement fy, sections width b, and unit cost of steel to that of concrete q. The design output includes: optimum area of reinforcement As_{opt}, and optimum effective depth of section d_{opt}.

A set of 21691 and 12555 optimum design examples were generated for training and a set of 1491 and 213 unseen examples were used for testing of trained ANN for singly and doubly reinforced sections, respectively. Three layered feed forward neural networks (FFNN) consisting of one hidden layer has been simulated using MATLAB developed by [17] for learning of the optimal design examples .The range of input and output data are shown in Table 4.

Table 4. Range of input and output parameters in database for the optimum designs SRB-DRB

	Singly reinforcement		Doubly reinforcement	
Input parameter	**Minimum**	**Maximum**	**Minimum**	**Maximum**
Width (mm) *b*	**200**	**400**	**200**	**400**
Compressive strength (MPa) fc	**20**	**40**	**20**	**40**
yield strength (MPa) fy	**300**	**520**	**300**	**520**
cost of steel/concrete	**15**	**95**	**10**	**35**
Ultimate moment(kN-m)	**100**	**2000**	**150**	**1675**
Area of steel (mm^2) A_s	**648**	**10613**	**947.7**	**4499.3**
Depth (*mm*) *d*	**301**	**1100**	**301.2**	**961.4**
Area of positive steel(mm^2)			**100.2**	**2189.7**

The multi-layer feed forward back-propagation technique [18] is implemented to develop and train the neural network of current study where the sigmoid transform function is adopted. The term "ANN prediction" is reserved for ANN response for cases that were not used in the pre-training stages. This is used in order to examine the ANN's ability to associate and generalize a true physical response that has not been previously "seen." A good prediction for these cases is the ultimate verification test for the ANN models. These tests have to be applied for (input and output) response within the domain of training. It should be expected that ANN would produce poor results for data that are outside the training domain.

Preprocessing of data by scaling was carried out to improve the training of the neural network. To avoid the slow rate of learning near the end points specifically of the output range due to the property of the sigmoid function, the input and output data were scaled between the interval 0.1 and 0.9. The scaling of the training data sets was carried out using the following equation:

$$y = (0.8/\Delta)x + (0.9 - 0.8x_{\max}/\Delta) \tag{51}$$

where $\Delta = x_{\max} - x_{\min}$

It should be noted that any new input data should be scaled before being presented to the network and the corresponding predicted values should be un-scaled before use. The back-propagation learning algorithm was employed for learning in the MATLAB program [17]. Each training "epoch" of the network consisted of one pass over the entire all training data sets. The testing data sets were used to monitor the training progress.

Different training functions available in MATLAB were experimented for the current application. The Levenberg-Marquardt (LM) techniques built in MATLAB proved to be efficient training functions, and therefore, are used to construct the NN model. These training functions are among the conjugate gradient algorithms that start training by searching in the steepest descent direction (negative of the gradient) on the first iteration.

The LM algorithm is known to be significantly faster than the more traditional gradient descent type algorithms for training neural networks. It is, in fact, mentioned as the fastest method for training moderately sized feed-forward neural network [19]. While each iteration of the LM algorithm tends to take longer than each iteration of the gradient descent algorithm used previously, the LM algorithm yields far better results using far fewer iterations, leading to a net saving in computer processor time over the previous method. One concern, however, is that it may overfit the data. The network should be trained to recognize general characteristics rather than variations specific to the data set used for training.

The network architecture or topology is obtained by identifying the number of hidden layers and the number of neurons in each hidden layer. There is no specific rule to determine the number of hidden layers or the number of neurons in each hidden layer. The network learns by comparing its output for each pattern with a target output for that pattern, then calculating the error and propagating an error function backward through the neural network. To use the trained neural network, new values for the input parameters are presented to the network. The network then calculates the neuron outputs using the existing weight values developed in the training process. Table 5 shows the properties (architectures and parameters) of ANN models.

Table 5. Properties of ANN models

	Singly reinforcement model	Doubly reinforcement model
Architecture	5-12-2	5-15-3
training function	LM	LM
Activation Function	Log sigmoid	Log sigmoid
Mean Squared Error (MSE)	0.0005	0.0005

6.3 Results and Discussion

The performance of a trained network can be measured to some extent by the errors on the test sets, but it is often useful to investigate the network response in more detail. One option is to perform a regression analysis between the network response and the corresponding targets and finding a correlation coefficient. It is a measure of how well the variation in output is explained by the targets. If this number is equal to 1, then there is perfect correlation between targets and outputs.

The regression analysis between the ANN predicted and corresponding calculated optimum values for depth and steel area are shown in Figs. (8) to (12), the correlation coefficients were found to be 0.98769 and 0.99578 for the depth of singly and doubly reinforced section, respectively, while the correlation coefficients for tension steel area were 0.99416 and 0.99438 for singly and doubly reinforced sections, respectively. On the other hand the correlation coefficient for compression steel area of doubly reinforced sections was 0.99026.

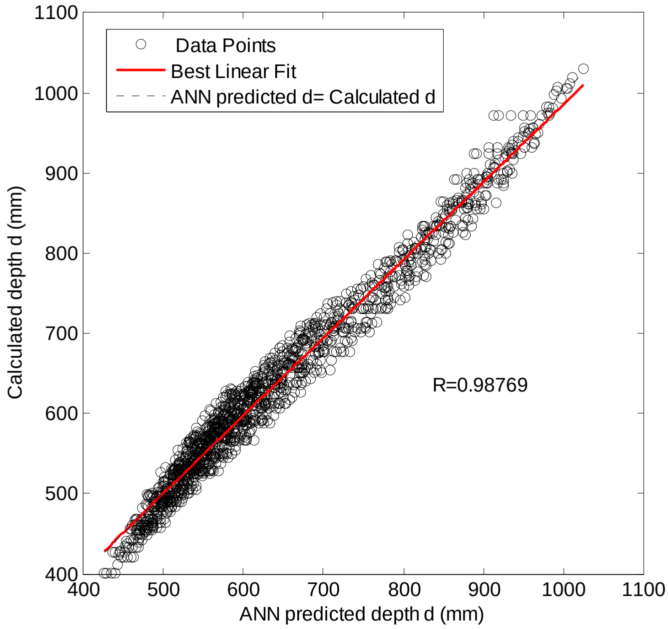

Fig. 8. Calculated and predicted depth of regression for test data of SRB

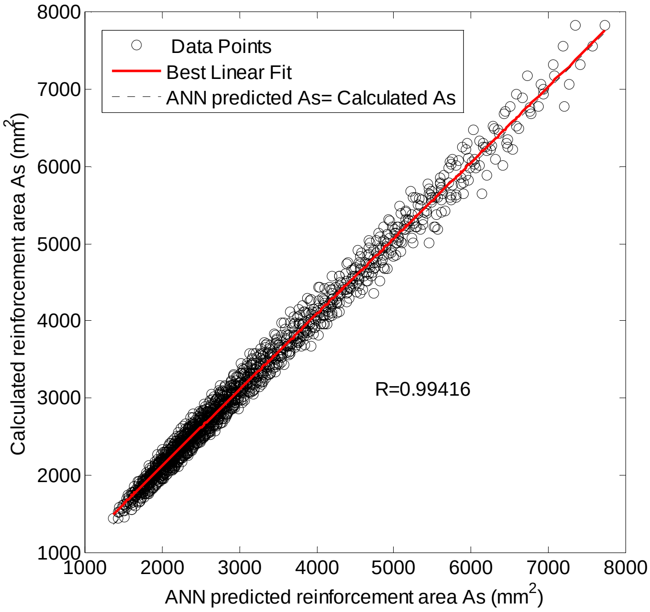

Fig. 9. Calculated and predicted reinforcement area of regression for test data of SRB

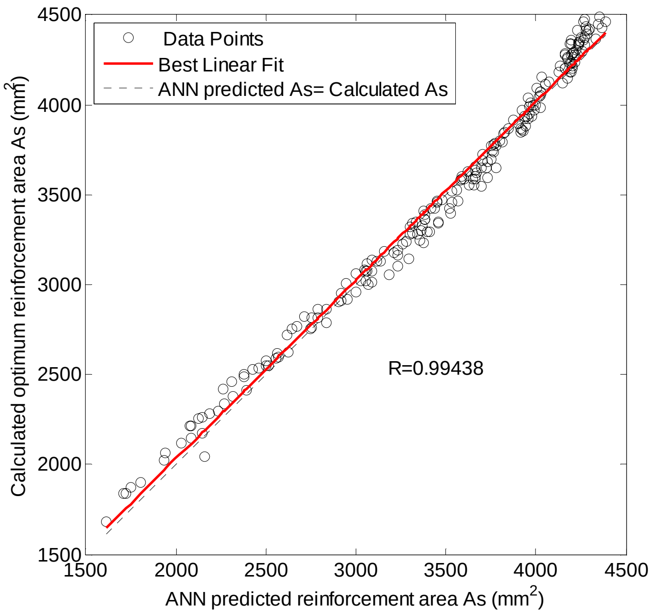

Fig. 10. Calculated and predicted depth of regression for test data of DRB

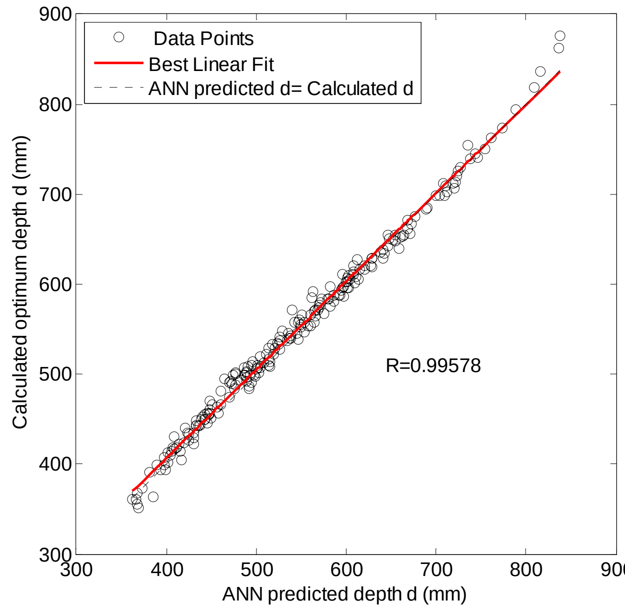

Fig. 11. Calculated and predicted reinforcement area of regression for test data of DRB

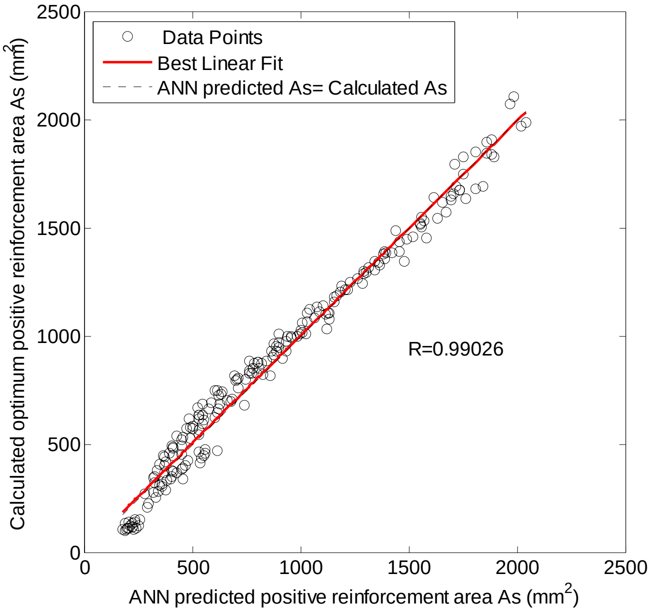

Fig. 12. Calculated and predicted compression reinforcement area of regression for test data of DRB.

It is clear that neural network provides an efficient alternative method in the design of singly and doubly reinforced concrete beam sections.

The neural network approach was adopted in an attempt to overcome significant limitations with traditional methods. Compared to similar works using the ACI method, the neural network approach does not require any equations; all the user has to do is input a few parameters describing the specific problem to be solved. In addition, a neural network can solve simultaneously a batch of problems in almost negligible time.

The success of the ANN model in predicting the design parameters highlights that such a numerical technique can be used reliably to design problems for structural elements.

7. Conclusions

In this work, the optimum design of SRB and DRB was done by taking moment-equilibrium besides other constrains. To evaluate the cost of the beam, a ratio of steel to concrete costs is necessary. Two design variables ρ and R, and other factors are used, and the optimum design problem can be solved easily using LMM without need for iterative trials. The artificial neural networks (ANN) has been trained with design data obtained from optimal design formulas. After successful learning, the model predicted the depth of the beam section and area of steel required for problems.

The research reported in this paper shows the following conclusions:

- The optimum steel ratio ρ_{opt}, is usually less than ρ_u and considerably greater than ρ_1.
- The optimum section is very economical as compared to other sections which can be obtained from standard design method.
- The procedure developed can serve as the basis for designing reinforced concrete beams, while a structure using the optimum section will not provide an optimum design for the entire structure.
- The problem has been limited about the singly reinforced beam section, if q and $f'c$ are relatively small and f_y is large, it appears possible that the doubly reinforced section could be the optimum section.

- The feasibility of using the artificial neural networks in building the model for optimum design of SRB and DRB, has been verified, the artificial neural network model predicted the optimum depth of the beam sections and optimum areas of steel required for the problems with accuracy satisfying all design constraints.

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Notation

The following symbols are using in this paper

a	depth of the compression stress block	R	Correlation coefficient
A_S	Area of tensile steel reinforcement	R_1	coefficient for ρ_u
A_{sopt}	optimum tension steel area	R_u	coefficient for $\rho 1$
A'_S	Area of compression steel reinforcement	R^m_{opt}	optimum coefficient without steel limit constraint
$A'_{s\,opt}$	optimum compression steel area	t	dimensionless geometrical properties of rectangular beam section (see Eq. 7)
b	width of beam	V_c, V_S	volumes of concrete and steel of beam of unit length
c	Distance from top fiber to natural axes	w	objective function
C	cost of unit length of beam, i.e., cost of section	β_1	equivalent stress factor
C_c, C_s	costs of concrete and steel per unit volume ,respectively	λ	Lagrange multiplier
d	effective depth (to tension reinforcement)	ρ	$As/b\,d$
d_{opt}	optimum effective depth	$\rho 1$	minimum reinforcement ratio
d_s	distance from steel centroid to tensile face	ρ'_{cy}	minimum tensile reinforcement ratio that will ensure yielding of the compression steel at failure
f'_c	strength of concrete	ρ_{doubly}	tension reinforcement ratio for doubly reinforced section
f_y	yield strength of steel	$\rho'_{\max}$	maximum tension reinforcement ratio for doubly reinforced section
g	constrains function	ρ_{opt}	optimum tension reinforcement ratio
k_n	Flexural resistance factor $= \rho f_y (1 - 0.59 \frac{\rho f_y}{f'_c})$	ρ^m_{opt}	optimum tension reinforcement ratio without steel limit constraints
		ρ'_{opt}	optimum compression reinforcement ratio
L	Lagrange function	ρ'	$A's/b\,d$
M_n	nominal bending moment	ρ_u	maximum tension reinforcement ratio
M_u	ultimate bending moment	ϕ	strength reduction factor see Fig.(2)
q	ratio of cost of steel to that of concrete	ε_t	net tensile strain of steel
Q	(1+t)	ε_y	yield strain of steel (f_y / E_y)
R	coefficient of $\sqrt{M_n / b}$	ε_u	ultimate strain of concrete

Abbreviations:

ANN: Artificial Neural Network
FFNN: Feed Forward Neural Networks
SRB: Singly Reinforced Beam
DRB: Doubly Reinforced Beam
LMM: Lagrange Multiplier Method