Estimated Equations for Water Flow Through Packed Bed of Mono Size Spherical Packing System

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Abstract

Semi-empirical equation for water flow through packed bed of sphere particles of mono size packing system has been estimated depending on Buckingham π theorem. Different parameters affecting the pressure drop of fluid flow through packed bed have been studied. These parameters are fluid velocity, bed porosity, bed diameter, sphericity, particle diameter, packing height and wall effect. Several types and kinds of packing materials have been used in this study such as (Pea Gravel, Marbles, Glass Marbles, Black Marbles, Clear Marbles, Acrylic balls and Glass spheres). The diameters of the packing materials used in this model are from the range of (0.2-8.89) cm, the porosity is from the range of (0.3-0.47), the bed diameters is from the range of (7.62 - 15.24) cm and the height of packing is from the range of (26.03 - 55.88) cm.

The results of all calculations for the estimated equations have been compared with many documented experimental literatures. This comparison gave a very good agreement, and has been represented in curves. The results from Ergun equation using similar conditions have been represented in the curves for the sake of comparison.

معادلات مقترحة لجريان الماء خلال العمود الحشوي لحشوات احادية كروية الشكل

نغم عبيد كريم

الخلاصة

معادلات شبه عملية لجريان الماء خلال عمود حشوي لحشوات كروية الشكل ذات حجم واحد تم صياغتها بالاعتماد على نظرية باكنكهام. تم دراسة العوامل المختلفة التي تؤثر على هبوط الضغط عند جريان الموائع في العمود حشوي، هذه العوامل هي سرعة جريان الموائع، مسامية الحشوة، طول الحشوة في العمود الحشوي، قطر العمود الحشوي، معامل كروية الحشوات، قطر الحشوة ودراسة تاثير جدار العمود الحشوي. تم استخدام انواع واشكال مختلفة من الحشوات وبحجوم مختلفة. اقطار الحشوات المستخدمة بهذه المعادلة هي تتراوح بين (0.2 الى 8.89 سم) , مسامية الحشوة تتراوح بين (0.3 الى 0.47)، قطر العمود الحشوي يتراوح بين (7.62 الى15.24 سم) وطول العمود الحشوي يتراوح بين (26 الى 55.88 سم).

النتائج المستحصلة من حسابات المعادلات المصاغه تم مقارنتها مع عدد كبير من النتائج العملية المستحصلة من المصادر الموثقة،و هذه المقارنة اعطت تطابق جيد جدا، وتم عرض ذلك في رسومات. تم وضع النتائج المستحصلة باستخدام معادلة ايرجن في هذه الرسومات لنفس الظروف السابقة لغرض المقارنة.

1. Introduction

Fluid flow through packed bed is a frequent occurrence in the chemical industry and therefore expressions are needed to predict pressure drop across beds due to the resistance caused by the presence of the particles [1].

A typical packed bed setup is a cylindrically-shaped column filled with packing materials. The column can vary in diameter, height, and material. The packing material can vary in shape, roughness, and particle size [2, 3]. The most important factor in concerning the bed from a

mechanical perspective is the pressure drop required for the liquid or the gas to flow through the column at a specified flow rate [4].

The fluid flow through packed bed has attracted considerable attention from many investigators. Darcy [5] derived a semi-empirical equation describing fluid transport in packed bed for single-phase flow. Carman and Kozeny [6] derived an expression for pressure drop under viscous flow. Burke and Plummer [7] derived an expression for change in pressure at turbulent flow resulting from kinetic energy .Ergun [8] proposed a semi-empirical equation covers any flow type and condition (laminar, transitional and turbulent) by adding the Carman-Kozeny equation for laminar flow to the Burke-Plumner equation derived for the fully turbulent. Ergun equation applies to a broad spectrum of fluids and packing materials, but it does not predict pressure drop behaviour after the point of fluidization because of bed expansion and changes in packing void fraction [9]. Ergun's equation does not take in consideration wall effects, which represents pipe like flow around the edges of the column [10, 11]. Leva [12] predicted the pressure drop of flow rate based on the study of single incompressible fluids through an incompressible bed of granular salts. Dullien and MacDonald addressed the problem of multi-sized particles present in a porous media. Dullien [13] modified Kozeny equation assuming pores with periodic step changes in their diameter. MacDonald [14] generalized the Blake-Kozeny equation for multi sized spherical particles. Bey and Eigenberger [15] have represented the pressure drop in the packing by modifying the Ergun equation for a cylindrical coordinated system. Shenoy *et al.* [16] developed a theoretical model for the prediction of velocity and pressure drop for the flow of a viscous power law fluid through a bed packed with uniform spherical particles. Hellström and Lundström [17] suggested a model for flow through packed bed taking into consideration the inertia-effects. They compared their results with Ergun equation, and it fits well to Ergun equation.

2. Theoretical Part: Water Flow Semi-Empirical Equations

Semi-empirical formulas for modelling water flow through packed bed were estimated for the parameters affecting the pressure drop using Buckingham π theorem [18]. This formula consists of multiplied dimensionless terms raised to certain powers [19]; these powers were evaluated from experimental data taken from literatures with statistical fitting.

The method of modelling used to derive an expression for the pressure drop was based on curve fitting of the available literatures experimental data, and by implementing dimensional analysis. This analysis can be summarized as follows:

The pressure drop was assumed to be dependent on fluid velocity (u),packing diameter (d_p),bed length (L),fluid density (ρ),fluid viscosity (μ),porosity (ε),and sphericity (ϕ), and can be written in the following expression

$$\Delta P = f\left(u, d_p, L, \rho, \mu, \varepsilon, \phi\right) \tag{1}$$

The Buckingham's π theorem [18] was used to write the semi-empirical formula of the fluid flow equation. In this theorem the dimensions of a physical quantity are associated with mass, length and time ,represented by symbols M, L and T respectively, each raised to rational powers [20]. The Buckingham's π theorem [18] forms the basis of the central tool of the dimensional analysis. This theorem describes how every physically meaningful equation involving n variables can be equivalently rewritten as an equation of *n-m* dimensionless parameters, whereas, the number of fundamental dimensions used. Furthermore, and the most important is that it proves a method for computing these dimensionless parameters from the given variables [19]. According to this theorem n=8 and m=3, then this theorem gave us five dimensionless groups.

Table 1. The Dimensions of Parameters Used in Expression (1).

Variable	Dimension
Pressure drop	$M L^{-1} T^{-2}$
Fluid velocity	$L T^{-1}$
Particle diameter	L
Bed length	L
Fluid density	$M L^{-3}$
Fluid viscosity	$M L^{-1} T^{-1}$
Porosity	-
Sphericity	-

Selecting the variables particle diameter, fluid velocity, and fluid density.

The particle diameter (d_p) has the dimension L therefore $L = d_p$

The fluid velocity (u) has dimensions $L T^{-1}$ therefore $T = d_p u^{-1}$

The fluid density (ρ) has dimensions $M L^{-3}$ therefore $M = \rho d_p^{3}$

The first group $(\pi_1) = \Delta P (M^{-1} L T^2)$

$$\pi_1 = \frac{\Delta P}{\rho u^2} \tag{2}$$

The second group $(\pi_2) = L (L^{-1})$

$$\pi_2 = \frac{L}{d_p} \tag{3}$$

The third group $(\pi_3) = \mu (M^{-1} L T)$

$$\pi_3 = \frac{\mu}{\rho u d_p} \tag{4}$$

The fourth group $(\pi_4) = \varepsilon$

$$\pi_4 = \varepsilon \tag{5}$$

The fifth group $(\pi_5) = \phi$

$$\pi_5 = \phi \tag{6}$$

Therefore the equation for the pressure drop according to our assumption dependence on fluid velocity (u), packing diameter (d_p), bed length (L), fluid density (ρ), fluid viscosity (μ), porosity (ε), and sphericity (ϕ) will be as follows

$$\frac{\Delta P}{\rho u^2} = b_1 \left(\frac{L}{d_p} \right)^{b_2} \left(\frac{\mu}{\rho d_p u} \right)^{b_3} \varepsilon^{b_4} \phi^{b_5} \tag{7}$$

while Reynolds number is defined as

$$\text{Re} = \frac{\rho d_p u}{\mu} \tag{8}$$

Then equation (7) can be written as follows

$$\frac{\Delta P}{\rho u^2} = b_1 \left(\frac{L}{d_p} \right)^{b_2} \left(\frac{1}{\text{Re}} \right)^{b_3} \varepsilon^{b_4} \phi^{b_5} \tag{9}$$

where b_1, b_2, b_3, b_4 and b_5 are constants which can be evaluated from experiments data taken from literature by statistical fitting. The above equation can be used for different types of packing system.

Since ($\Delta P / \rho u^2$) describes fluid flow through packed bed, therefore; equation 9 can be considered as a semi-empirical equation of fluid flow through packed bed. Each term of this equation is a dimensionless group, because ($\Delta P / \rho u^2$) is dimensionless number.

3. Results and Discussion

The results of the estimated new semi-empirical equations are presented, discussed and compared with experimental results taken from literatures, as well as with results taken by using Ergun equation for water flow through packed bed.

Ergun believed that the pressure drop over the length of the packing was dependent upon rate of fluid flow, viscosity and density of the fluid, closeness and orientation of packing, size, shape, and surface of the packing material [21, 22]. **Ergun** equation can be expressed as:

$$\frac{\Delta P}{L} = 150 \ \frac{\mu u}{\phi^2 d_p^2} \frac{(1-\varepsilon)^2}{\varepsilon^3} + 1.75 \ \frac{\rho u^2}{\phi d_p} \frac{(1-\varepsilon)}{\varepsilon^3} \tag{10}$$

where ΔP, ε, ρ, d_p, Φ, u, L, and μ are the pressure drop, void fraction of the bed, density of the fluid, particle diameter, sphericity of the particle, fluid velocity, height of the bed, and the fluid viscosity respectively.

The estimated semi-empirical equation (9) was fitted for water flow through packed beds of mono size spherical particles system. In this fitting 40 sets of data from the literatures [3, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34,35, 36, 38, 39, 40, 41, 42, 43, 44, 45] were used. In these sets 396 values of pressure drop versus velocity were taken. Several types and kinds of packings have been used such as (Pea Gravel, Marbles, Glass Marbles, Black Marbles, Clear Marbles, Acrylic balls and Glass spheres). The diameters of the packing materials used are from the range of (0.2-8.89) cm, the porosity is from the range of (0.3-0.47), the bed diameters is from the range of (7.62 - 15.24) cm and the height of packing is from the range of (26.03 - 55.88) cm. So the new fluid flow model for water flow through packed beds will be as follows

$$\frac{\Delta P}{\rho u^2} = 230840.9 \ \left(\frac{L}{d_p}\right)^{0.83} \left(\frac{1}{\text{Re}}\right)^{0.88} \varepsilon^{-3.03} \ \phi^{0.1} \tag{11}$$

The average percentage errors were found to be 4.83% between experimental work and the estimated equation model.

Most of the previous experimental works were in the terms of pressure drop. So for the sake of comparison with the available experimental data, equation 11 has been multiplied by (ρu^2).The new form of the equation will be a pressure drop equation.

The results of the estimating equation, eq. (11), for mono size spherical packing system are presented in this section. This presentation takes into account a comparison between these results and experimental results, as well as comparisons were made between all these results and similar results taken by using Ergun equation for water flow through packed bed. These comparisons are shown in figures below
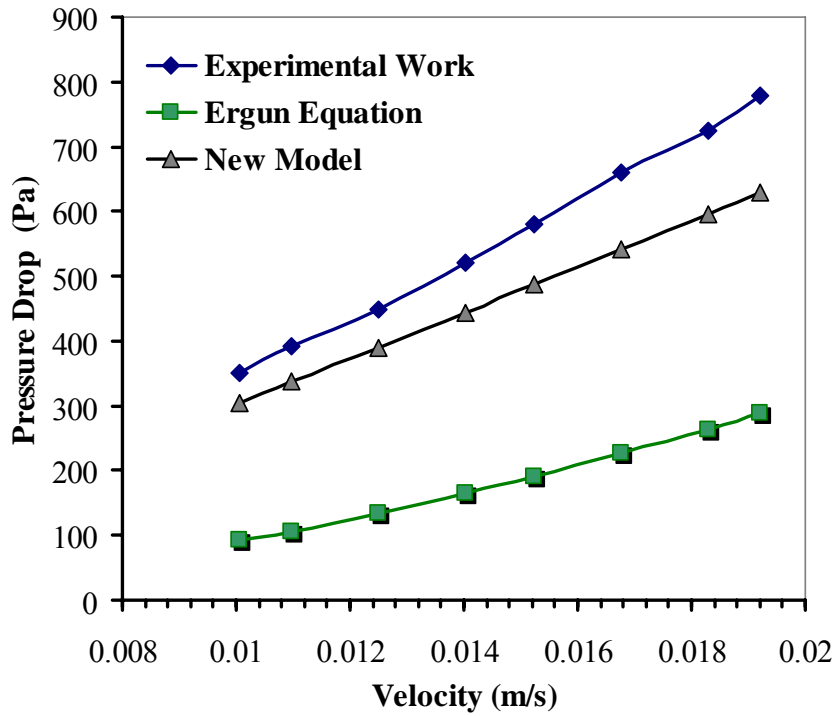
Fig. 1. Pressure drops versus velocity for Pea Gravel of diameters 1.27 cm, bed porosity of 0.38, packing height of 41.28 cm, and bed diameter of 8.89 cm [34].
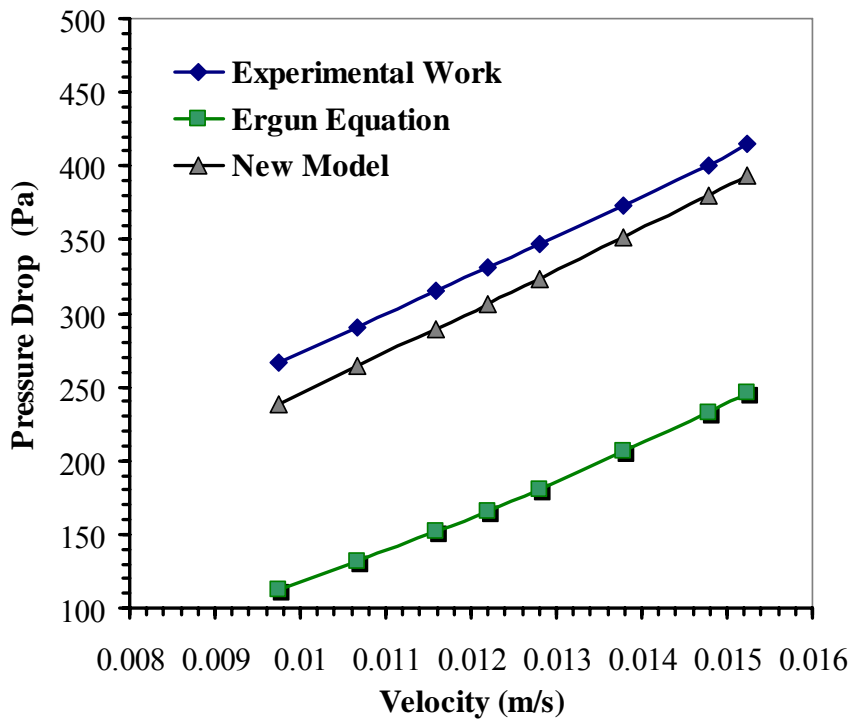
Fig. 2. Pressure drops versus velocity for Pea Gravel of diameters 1.27 cm, bed porosity of 0.393, packing height of 53.34 cm, and bed diameter of 8.89 cm [38].
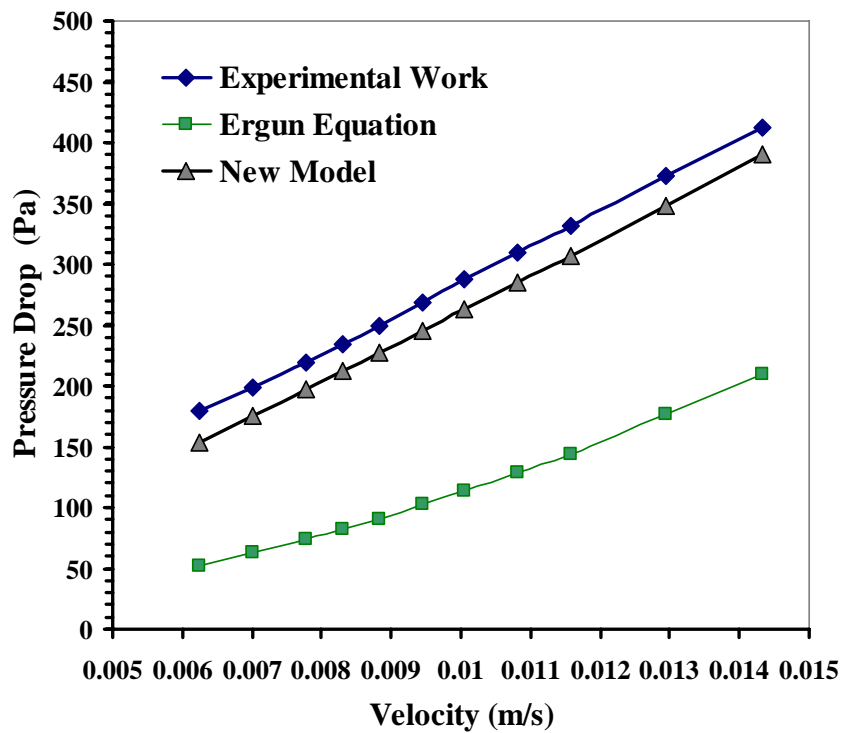
Fig. 3. Pressure drops versus velocity for Pea Gravel of diameters 1.27 cm, bed porosity of 0.3902, packing height of 50.8 cm, and bed diameter of 15.24 cm [39].

The above figures show clearly that the estimated equation results of pressure drop-velocity curves are coincide with experimental results, while the results from Ergun equation lie below them; this may be due to:

1. Differences in beds dimensions, packing shapes and sizes used by Ergun [8, 9].
2. Ergun designed his equation using completely different procedures than experimental data work [38, 8].
3. This was expected because of changing the properties of the packing materials leads to large effect on Ergun equation's prediction of pressure drop [34].
4. Other reasons of this large deviation from Ergun equation, that Ergun's equation does not take in to consideration wall effects [25].

The wall affect on bed porosity increases the porosity, this appears clear in Figures 1 and 2 where the bed porosity increased to a value of 0.38, while the bed diameter is 8.89 cm, and the particle diameter is 1.27 cm. The effect of wall on porosity may be due to the reduction in the ratio of bed diameter to particle diameter than the supposed ratio $((D_R/d_P) \geq 10)$ [40].

From the above figures it can be seen that as the packing height increases the pressure drop increases, this is because when the packing height increases the fluid flow resistance increases and this leads to an increase in the pressure drop.

4. Conclusions

Comparing the results of the estimated equations of pressure drop versus velocity curves with those of experimental data and Ergun equation results; it indicates that the estimated equations results coincide with experimental results, while the results from Ergun equation was far away from them.

It was found that an increase in particle diameter causes a decrease in pressure drop, this is because when the particle diameter increase's the specific surface area of it decreases, and this leads to a decrease in the resistance to fluid flow.

The bed porosity highly affects the pressure drop and inversely proportional to it, this is because that when the porosity increases the resistance to fluid flow through the bed decreases.

It was found that the pressure drop through a packed bed is highly sensitive to the packing height and that as the packing height increases the pressure drop increases.

References

1. J. Paul, Sides, "Theory of Pressure Drop for Flow in Packed Beds". CMU Spring 2004.

2. G. Pan and H. Yun, "Flow in Packed Beds". Carnegie Mellon University (2005). Article given on the internet at the web site:
    http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t5_s04/t5_s04.pdf

3. A. Edison and J. Kim, "Correlation of Pressure Drop and Flow Rate Using Packed Beds". Carnegie Mellon University, Chemical Engineering Department (2006). Article given on the internet at the web site:
    http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t8s06/t8_s06_r1s2.pdf

4. M. Dalka and K. Evans, "Packed Beds: Pressure Drop and Friction Factors with Varying Packing Material, Column Diameter and Flow Rate", (2004). Article given on the internet at the web site:
    http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t1_s04/t1_s04.pdf

5. H. P. G .Darcy, 1856, Les Fontaines Publiques de la ville de Dijon., Dalmont, Paris.

6. J. Kozeny, Ber, Wien Akad, 136 (1927) 271.

7. S. P. Burke and W. B. Plummer, "gas flow through packed column". Ind. Eng. Chem. Vol. 20, 1928, pp. 1196-1200.

8. Ergun, Sabri. "Fluid Flow through Packed Columns", Chemical Engineering Progress, vol. 48, pp. 89-94. American Institute of Chemical Engineers: 1952, New York.

9. Ergun, Sabri. "Determination of Size Distribution of Macropores in Porous Materials". Carnegie Institute of Technology. 1953.

10. J. M. Coulson and J.F. Richardson, "Chemical Engineering", volume II, Third Edition, Pergamon Press, London, 1985.

11. R. K. Niven, "Physical insight into the Ergun and Wen and Yu Equations for Fluid Flow in Packed and Fluidized Beds," Chem. Eng. Sci. Vol. 57, No.3, pp.527–534, 2002.

12. M. Leva, Fluidization, Mc Graw –Hill, New York, 1959.

13. F. A. L. Dullien, M. S. El- Sayed, and V. K. Batra, "Rate of Capillary Rise in Porous Media with Non Uniform Pores", Journal of Colloids and Interface Science, **60**, 497-506 (1976).

14. M. J. MacDonald, C. Chao - Feng, P. P. Guilloit and K. M. Ng, "A generalized Blake-Kozeny equation for Multi sized Spherical Particles", AIChE Journal, 37, 1583 (1991)

15. O. Bey and G. Eigenberger, 1997, "Fluid Flow Through Catalyst Filled Tubes", Chemical Engineering Science, 52(8), pp. 1365-1376.

16. A. V. Shenoy, A. Afacan, B. Boulanger and R. E. Hayes, "Modelling the flow of power law fluids in a packed bed using a volume-averaged equation of motion". J. Powder technology, Vol. 23, No. 2, 175-196, 1996.

17. J. G. I. Hellström, T. S. Lundström," Flow through Porous Media at Moderate Reynolds Number", International Scientific Colloquium Modeling for Material Processing, Riga, June 8-9, 2006.

18. Buckingham, Edgar (1914). "On Physically Similar Systems: Illustrations of the Use of Dimensional Analysis". Phys. Rev. 4: 345.

19. Dimensional analysis-Wikipedia, the free encyclopedia. Article given on the internet at the web site: http://en.wikipedia.org/wiki/Dimensional_analysis

20. Dimensionless quantity-Wikipedia, the free encyclopedia.Article given on the internet at the web site: http://en.wikipedia.org/wiki/Dimensionless_quantity

21. E. Saw and A. Yang, "Pressure Drop for Flow through Packed Beds", (2004). Article given on the internet at the web site:
http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t4_s04/t4_s04.pdf

22. Geankoplis, Christie J., Transport Process and Unit Operations. 4th ed.,   New Jersey: Prentice Hall, 2003.

23. S. Back, A. Beaber, E. Boudreaux and K. Paavola," Pressure Drop for Flow in Packed Beds: An analysis using Ergun's Equation",(2004).Article given on the internet at the web site:
http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t3_s04/t3_s04.pdf

24. P. Chung, R. Koontz, and B. Newton, "Packed beds: Pressure drop versus fluid velocity and the Ergun equation" (2002). Article given on the internet at the web site:
http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t10_s02/t10_s02.pdf

25. L. Kovell and M. Jordan. "PACKED BEDS". Carnegie Mellon University, (2007). Article given on the internet at the web site:
http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t2_s05/r2/t2_s05.pdf

26. M. Chopard and A. Welsh, "Packed beds and Ergun equation: the relationship between fluid flow and pressure drop", (2003). Article given on the internet at the web site:
http://rothfus.cheme.cmu.edu/tlab/pbeds/pbeds.PDF

27. H. R. Salah, M.SC. Thesis, "Characterization of Mono, Binary And Ternary Sphere Packing", Nahrain University, 2007.

28. W. A. Abd Al-Nabi, M.SC. Thesis, "Characterization Of Single And Multi Sizes Sphere Packing", Nahrain University, 2007.

29. Z. S. AL-Rawi, M.SC. Thesis, "Air Flow through Packed Bed for Multi–Size Particles", Saddam University, 2001.

30. E. Miller and P. Shah, "Fluid Flow though Packed Beds", (2001).   Article given on the internet at the web site:
http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t8s06/Appendices.zip

31. M. Dileo and J. Hung, "Fluid Flow in Packed Beds: A comparison of Ergun and Leva Equations", (2005). Carnegie Mellon University, Chemical Engineering Department. Article given on the internet at the web site:
http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t7_s05/t7_s05.pdf

32. M. Chopard and A. Welsh, "Packed Beds", (2007). Article given on the internet at the web site:
http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t9s06/PackedBedPresentation_t9s06.ppt

33. S. Dhanani, R. Gillani and N. Shedd, "Flow through Packed Beds",(2006). Mellon University, Chemical Engineering Department. Article given on the internet at the web site:
http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/ /t4s06/t4_s06_r1s2/t4_s06_r1s2.pdf.

34. S. Basu, C. Dixon and C. Fereday, "Comparison of the Ergun Equation with Experimental Values Regarding Pressure Drop and Fluid Velocity", (2003). Article given on the internet at the web site:
http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t2_s03/t2_s03.pdf

35. J. Boss and H. Lim, "Packed beds: Predicting pressure drops with two different sizes of spheres", (2001). Mellon University, Chemical Engineering Department .Article given on the internet at the web site:
http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t7_s01/t7_s01.pdf

36. L. Brown and S. Carothers, "Non-fluidized Packed Beds and the Effect of Packing Surface Properties", (2001). Carnegie Mellon University. Article given on the internet at the web site: http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t1_s01/t1_s01.pdf

37. S. Dence and H. Leifeste, "Packed Beds: An Improvement to a Classical Model", (2001). Article given on the internet at the web site: http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t2_s01/r1s2/t2_s02.pdf

38. C. Pierce and J. Williams, "Study of Pressure Drop vs. Flow Rate in Packed Beds", (2005). Article given on the internet at the web site: http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t6_s03/t6_s03.pdf

39. Z. Britton and T. Donegan. "Packed Beds Experiment". Carnegie Mellon University, Chemical Engineering Department (2003).Article given on the internet at the web site: http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t9_s03/t9_s03.pdf

40. C. Pierce, J. Williams and R. Milloy, "Study of pressure drop versus flow rate in packed beds", (2002). Article given on the internet at the web site: http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t5_s02/t5_s02.pdf

41. R. Arffa, D. Chan and A. Shukla, "Examination of the Ergun Equation: Flow through Packed Beds". Article given on the internet at the web site: http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t9_s05/T09%20S05%20Packed%20Beds%20_Ergun_.pdf (2005)

42. K. Osayawe, R. Patel, "Investigation of the Friction Factor-Reynolds Number Relationship for Packed Beds", (2007). Article given on the internet at the web site: http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t7_s04/t7_s04.pdf

43. S. Chung and K. Henry, "Pressure Drop and Flow in Packed Beds: Comparison to the Ergun Equation", (2006). Article given on the internet at the web site: http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t9s06/t9_s06_r2.pdf

44. S. Hersman and B. Hunter, "Flow through Packed Beds: Investigation of the Ergun Equation and Development of a Straw Model of Packed Beds", (2006). Article given on the internet at the web site: http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t11_s05/t11_s05.pdf

45. M. Betler, C. Csider and G. Davenport, "Evaluation of the Validity of the Ergun Equation", (2002). Carnegie Mellon University, Chemical Engineering Department. Article given on the internet at the web site: http://rothfus.cheme.cmu.edu/tlab/pbeds/projects/t2_s02/t2_s02.pdf

Notation

A	=	The bed cross sectional area (m).
D_R	=	Diameter of the bed (m).
d_p	=	Diameter of the particle (m).
dp_{eff}	=	Effective particles diameter (m).
d_{pi}	=	Diameter of particle i in mixture (m).
e	=	Porosity of the bed.
K	=	Dimensionless constant whose value depends on physical properties of the bed and fluid.

L	=	The height of packing in the bed (m).
l	=	Thickness of the bed (m).
ΔP	=	Pressure drop through packed bed, Pa (kg/m.s^2).
R	=	Reduce of horizontal pipe.
Re	=	Reynolds number.
u	=	Superficial velocity (m/s).
V	=	Volume of the fluid flowing through bed in time *t*.
x_i	=	The weight fraction of particle *i*.

Greek Symbols

ε	=	Porosity of the bed.
μ	=	Fluid viscosity (kg/m.s).
Φ	=	Sphericity.
ρ	=	Density of fluid (kg/m^3).
ρ_p	=	Density of particle (kg/m^3).
ρ_b	=	Bulk density (g/cm^3).
ρ_t	=	True density (g/cm^3).